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Abstract

We develop a unified framework at the crossroads of economics, political and environmental science, and, to some extent, epidemiology. Populism is equated with climate skepticism and seen as an opinion that spreads through the population. Drawing on compartmental models in epidemiology, the population is divided into two groups that interact with each other: climate skeptics, almost always populists, and environmentalists. The political building block is integrated into a Ramsey model with a pollution externality originated from production and impairing household's utility. We introduce a Pigouvian tax to finance depollution according to a balanced-budget rule. To take account of populist pressure against environmental policies, we assume also that the tax rate decreases in the share of skeptics in population. Our unified approach reveals an interesting result: populism generates stable limit cycles through a Hopf bifurcation around the steady state, whatever the pollution effect on the consumption demand. Thus, populism exacerbates pollution-induced volatility: populist parties focusing on economic issues should manage excess volatility without rejecting environmental policies out of hand.

Keywords: *ecotax, populism, political cycles.*
JEL codes: *C62, H23, O44.*

1 Introduction

According to Buzogány and Mohamad-Klotzbach (2021), both populism and climate change represent two major threats for contemporary democracies. While the definition of climate change is now well-established, populism remains more difficult to identify, as it encompasses both left and right. For Lockwood (2018) and Guriev and Papaioannou (2022), right- or left-wing populism are ideologies that divide the population into two groups: "the pure people" and a "corrupted elite", who oppose each other. However, as Huber (2020) points out,

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far-right parties reject environmental policies, while far-left parties broadly supports them.

Right-wing populism places great importance on economic issues rather than environmental ones (Lockwood, 2018). Two recent examples are Donald Trump in the USA, who withdrew the US from the Paris Agreement, and Jair Bolsonaro in Brazil, who authorized massive deforestation of the Amazon rainforest (Pereira et al., 2019). Buzogány and Mohamad-Klotzbach (2021) point out that right-wing populism is associated with anti-science, which partially explains why populists reject environmental policies. Two other explanations are proposed by Lockwood (2018): (1) his "structuralist" argument rests on fact that right-wing populism stems from the rejection of past structural changes such as globalization, and that climate change is seen as a new structural change that worsens the economic situation of those left behind, the so-called pure people populists seek to represent. The "ideological" explanation lies instead in the fact that right-wing populism is nationalist, and environmental concern is seen as a counter-national interest.

Over the past decade, right-wing populism has taken hold of the political landscape in European countries and the United States. The Brexit referendum in the UK in 2016, the elections of Donald Trump in 2017 and 2024 in the US, the recent electoral performances of the *Rassemblement National* in France or *Alternative für Deutschland* in Germany are all examples of this political breakthrough. It warns on the social acceptance of environmental policies.

The yellow-vests crisis in France is a good example of the possible social distrust of environmental policies: the massive protests that began in November 2018 convinced the French government to abandon the increase in the carbon tax on fuel. While the *gilets-jaunes* movement is clearly populist, it is far from clear whether it belongs to the right or the left (Bourdin and Torre, 2023). Nevertheless, from a theoretical point of view, it seems interesting to consider together populism and climate skepticism when discussing environmental policy. This article is a first attempt to introduce them in a dynamic general equilibrium context, while focusing only on right-wing populism: from now on, climate skepticism will be synonymous with right-wing populism.

To represent the evolution of public opinion, we propose to apply a simple model of disease spread.[1] Specifically, the population is divided into two groups: "environmentalists" and "skeptics". On each date, the agents meet and influence each other. During a bilateral encounter, a skeptic may become environmentalist and an environmentalist skeptic, depending on the relative power of persuasion. This power depends on the level of pollution: more (less) pollution gives credit to environmentalists (skeptics). We also assume that an agent can change her mind spontaneously. Our political model is close to the SIS epidemiological model but not identical.[2] Indeed, if we equate skeptics with sick

[1]We adapt Desmarchelier and Lanzi's (2023) opinion dynamics model to an environmental context.

[2]In the SIS model, infection does not confer long-term immunity and individuals become susceptible again when they recover from infection. The population is divided into labeled compartments: S and I stand for susceptible and infectives. Readers interested in a simple

agents and environmentalists with healthy agents, here, the latter can "contaminate" the former who, then, become environmentalists, whereas, in a SIS model, a healthy agent is unable to "contaminate" a sick agent.[3] In addition, the government is assumed to introduce a green tax levied on the production level to finance clean-up according to a balanced-budget rule. We suppose that the tax rate decreases with the share of skeptics in population: the more skeptics there are, the greater the pressure for a tax cut. This is what we call the Yellow-Vest Effect (hereafter YVE). This contagion model adapted to climate populism is integrated into a Ramsey model[4] to understand the interaction between economic variables (capital and consumption), opinion dynamics (climate skepticism/populism) and pollution dynamics.

Our model, at the crossroads of economics, political and environmental sciences and, to some extent, epidemiology, reveals that populism is not only bad for the environment, but also for economic stability. Populist parties that prioritize economic issues should manage excess volatility without rejecting environmentally-friendly solutions out of hand.

For the sake of precision, we know that limit cycles can appear via a Hopf bifurcation in a Ramsey model where pollution increases consumption demand (compensation effect), while they are ruled out when pollution lowers demand (distaste effect).[5] The rationale for the existence of cycles is quite simple: higher pollution today implies higher consumption demand (compensation effect) which reduces savings and, then, the capital stock of the next period. A lower capital stock decreases production possibilities and the stock of pollution in turn. And so on.

Let us now take into account the additional dynamics resulting from the spread of climate skepticism. The initially higher level of pollution strengthens the persuasive power of environmentalists who lobby to increase the ecotax rate and, ultimately, reduces production and pollution. So, now, in both cases, compensation or distaste effect with populism, cycles take place: higher pollution today is followed by lower pollution tomorrow, and so on.

We conclude by observing that not only does populism generate permanent cycles in the less favorable case of a distaste effect, where cycles are normally impossible, but it also exacerbates macroeconomic volatility in the more favorable case of a compensation effect, where cycles arise naturally. Populism implies that fluctuations generated by pollution can occur regardless of the effect of pollution on the marginal utility of consumption.

The remainder of the article is organized as follows. Section 2 presents the model, Section 3 studies the equilibrium, while Section 4 describes the local dynamics. Section 5 illustrates the analytical results through two simulations,

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presentation of this model are referred to Hethcote (2009).

[3]Social contagion has already been considered to explain how innovation spreads through a population (Young, 2009) or how cooperation takes place to manage a common resource (Richter et al., 2013).

[4]Ramsey (1928) is arguably the most influential model of economic growth. It was refined and popularized in 1965 by David Cass and Tjalling Koopmans.

[5]Distaste and compensation effects were introduced by Michel and Rotillon (1995). See also Bosi and Desmarchelier (2018).

while Section 6 concludes.

2 The model

We consider a Ramsey-type market economy where a productive pollution externality affects household utility. Agents have different ecological attitudes. Environmentalists are confronted with skeptics. The government levies a green tax to finance environmental maintenance and clean-up.

2.1 Producers

The production sector consists of a continuum of price-taker firms. Their output is produced according to a Constant-Returns-to-Scale (CRS) technology using capital and labor. Because of the CRS, it is equivalent to consider a single firm who behaves competitively:

$$Y \equiv F(K, L) = Lf(k), \tag{1}$$

where Y, K and L denote the aggregate supply of output and the aggregate demands for capital and labor, $k \equiv K/L$ and $f(k) \equiv F(k, 1)$ represent the capital intensity and the average output.

Assumption 1 $f''(k) < 0 < f'(k)$. *The Inada conditions hold:* $\lim_{k \to 0} f'(k) = \infty$ *and* $\lim_{k \to \infty} f'(k) = 0$.

The government levies a proportional tax on output at the rate $\tau \in (0, 1)$. This rate can vary over time: $\tau = \tau(t)$. Taking the announced tax rate as given at date t, the representative firm chooses the capital and labour demand to maximize its static profit: $\max_{(K,L)} [(1 - \tau) Lf(k) - rK - wL]$. Capital and labour demand are set to equalize marginal productivity and prices:

$$r = (1 - \tau) f'(k) = (1 - \tau) \rho(k), \tag{2}$$

$$w = (1 - \tau) [f(k) - kf'(k)] = (1 - \tau) \omega(k). \tag{3}$$

The capital share in total income is given by

$$\alpha(k) \equiv \frac{kf'(k)}{f(k)} \in (0, 1), \tag{4}$$

while the Allen-Hicks' elasticity of capital-labour substitution by

$$\varepsilon(K, L) = \frac{\frac{\partial F}{\partial K} \frac{\partial F}{\partial L}}{F \frac{\partial^2 F}{\partial K \partial L}}. \tag{5}$$

In the CRS case, this elasticity becomes:

$$\varepsilon(K, L) = -\frac{f'(k)[f(k) - kf'(k)]}{kf(k) f''(k)} \equiv \sigma(k) > 0, \tag{6}$$

and the elasticities of factor prices:

$$\frac{k\rho'(k)}{\rho(k)} = -\frac{1-\alpha(k)}{\sigma(k)} < 0 \text{ and } \frac{k\omega'(k)}{\omega(k)} = \frac{\alpha(k)}{\sigma(k)} > 0. \tag{7}$$

Under a constant elasticity of capital-labor substitution:

$$F(K,L) \equiv \left(aK^{\frac{\sigma-1}{\sigma}} + bL^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, \tag{8}$$

we get $\varepsilon(K,L) = \sigma(k) = \sigma$ and

$$\alpha(k) = \frac{a}{a + bk^{\frac{1-\sigma}{\sigma}}} \in (0,1). \tag{9}$$

In the Cobb-Douglas case, $\sigma = 1$ and α are constant.

2.2 Households

The representative household are represented by a continuous-time utility functional

$$\int_0^\infty e^{-\theta t} u(c(t), P(t))\, dt, \tag{10}$$

where θ denotes her time preference, u is a strictly increasing and concave, and non-separable felicity function depending on her consumption level $c(t)$ and an aggregate pollution externality $P(t)$.

Assumption 2 *$u_{cc} < 0 < u_c$ and $u_P < 0$ for any $(c,P) \in \mathbb{R}_+^2$. The Inada conditions hold: $\lim_{c\to 0} u_c = \infty$, $\lim_{c\to\infty} u_c = 0$.*

Remark 1 *The cross effect u_{cP} can be negative or positive under non-separability. Following Michel and Rotillon (1995), $u_{cP} < 0$ captures the distaste effect (a higher pollution level reduces consumption demand, namely when the household likes to consume in a pleasant environment). Conversely, $u_{cP} > 0$ represents a compensation effect (the household compensates the utility drop entailed by a higher pollution level ($u_P < 0$), by an increase of her consumption demand).*

Let $h(t)$ be the household's individual wealth in terms of capital, and $r(t)$ and $w(t)$ be the interest and wage rates. Let δ also be the constant rate of depreciation of capital. For simplicity, labor supply is inelastic and equal to one.

The household spends her income $r(t)h(t) + w(t)$ to consume and save $\dot{h}(t)+\delta h(t)$, where $\dot{h}(t)$ denotes the time derivative of individual wealth. For the sake of simplicity, we assume a constant population size over time: $N(t) = N$. The aggregate wealth is given by $H(t) = h(t)N$ and its variation by $\dot{H}(t)$. Thus, individual gross investment becomes

$$\frac{\dot{H}(t)}{N(t)} + \frac{\delta H(t)}{N(t)} = \dot{h}(t) + \delta h(t). \tag{11}$$

As usual in one-sector growth models, capital and consumption are the same good. Therefore, her budget constraint becomes

$$\dot{h}(t) = r(t)h(t) + w(t) - \delta h(t) - c(t). \tag{12}$$

From now on, for notational parsimony, we will omit the time argument. The household' program becomes

$$\max_{c} \int_{0}^{\infty} e^{-\theta t} u(c, P)\, dt, \tag{13}$$

$$\dot{h} = (r - \delta) h + w - c. \tag{14}$$

She chooses the entire path $c = c(t)$ in order to maximize this utility functional.

The first-order conditions of program (13) are given by

$$\frac{\dot{\mu}}{\mu} = \delta + \theta - r, \tag{15}$$

$$\dot{h} = (r - \delta) h + w - c, \tag{16}$$

where

$$\mu = u_c(c, P), \tag{17}$$

is the marginal utility of consumption.[6] The transversality condition becomes: $\lim_{t \to \infty} e^{-\theta t} \mu h = 0$.

We introduce two second-order elasticities of utility:

$$\varepsilon_{cc} \equiv \frac{c u_{cc}}{u_c} < 0 \text{ and } \varepsilon_{cP} \equiv \frac{P u_{cP}}{u_c}, \tag{18}$$

$-1/\varepsilon_{cc}$ represents the intertemporal elasticity of substitution in consumption while ε_{cP} captures the pollution effect on marginal utility of consumption. More precisely, in the spirit of Michel and Rotillon (1995) a compensation (distaste) effect takes place when $\varepsilon_{cP} > 0$ (< 0). Applying the Implicit Function Theorem to (17), we obtain the consumption function

$$c \equiv c(\mu, P), \tag{19}$$

with elasticities:

$$\frac{\mu}{c}\frac{dc}{d\mu} = \frac{1}{\varepsilon_{cc}} < 0, \tag{20}$$

$$\frac{P}{c}\frac{dc}{dP} = -\frac{\varepsilon_{cP}}{\varepsilon_{cc}}. \tag{21}$$

[6] Defining the Hamiltonian $H \equiv e^{-\theta t} u(c, P) + \lambda [(r - \delta) h + w - c]$, we obtain the first-order conditions: $\partial H / \partial c = e^{-\theta t} u_c(c, P) - \lambda = 0$, $\partial H / \partial \lambda = \dot{h}$ and $\partial H / \partial h = -\dot{\lambda}$, jointly with the transversality condition: $\lim_{t \to \infty} \lambda h = 0$. Redefining the multiplier: $\mu \equiv \lambda e^{\theta t}$, we get (15) to (17).

Focusing on elasticity (21), we observe that the impact of pollution on consumption is negative if $\varepsilon_{cP} < 0$ (distaste effect) and positive if $\varepsilon_{cP} > 0$ (compensation effect).

The elasticities of the explicit utility function:

$$u(c,P) \equiv \frac{(cP^{-\eta})^{1-\varepsilon}}{1-\varepsilon}, \tag{22}$$

with $\varepsilon > 0$ and $\eta > 0$, are constant:

$$\varepsilon_{cc} \equiv \frac{cu_{cc}}{u_c} = -\varepsilon < 0, \tag{23}$$

$$\varepsilon_{cP} \equiv \frac{Pu_{cP}}{u_c} = \eta(\varepsilon - 1) < 0 \Leftrightarrow \varepsilon < 1. \tag{24}$$

In this case, the impact of pollution on consumption is negative if $\varepsilon < 1$ (distaste effect) or positive if $\varepsilon > 1$ (compensation effect). We observe that η captures the degree of pollution externality. Since $\varepsilon_{cP} = \eta(\varepsilon - 1)$, it amplifies the compensation effect or the distaste effect.

Moreover, in the case of function (22), using (17), we obtain (19):

$$c(\mu,P) = \mu^{-\frac{1}{\varepsilon}} P^{\eta\frac{\varepsilon-1}{\varepsilon}}. \tag{25}$$

2.3 Government and pollution

Pollution P is a stock coming from production. To keep things as simple as possible, in the spirit of Keeler et al. (1971) among others, we assume a linear accumulation process:

$$\dot{P} = -aP + bY - dG, \tag{26}$$

where a, b, d and G represent, respectively, the rate of natural pollution absorption, the environmental impact per unit of production, the depollution efficiency and the depollution effort. In a world with no human (i.e. with $Y = G = 0$), from a date 0 on, pollution follows an exponential decay: $P(t) = P(0)e^{-at}$.

The government is responsible for environmental cleanup. More specifically, it uses all tax revenues to finance depollution:

$$G = \tau Y. \tag{27}$$

As usual in the literature, (27) means that the output can be used to depollute.[7] More precisely, (26) and (27) entail that the cleanup is linear with τY as only input.

[7] See, among others, Keeler et al. (1971) or Fernandez et al. (2012).

2.4 Political contagion

In the spirit of Desmarchelier and Lanzi (2023), a model of opinion dynamics, we represent the spread of skepticism through the economy. The population is divided in two groups, the "environmentalists" and the "environmental skeptics": $N = E + S$, where E and S denote the size of these two groups.

People can change their opinions. While, for simplicity, the size N of population is constant, the share $s \equiv S/N$ varies over time.

At any time, each agent interacts and exchanges opinions with another agent on climate change. The probability for a skeptic to meet an environmentalist is given by E/N. The number of skeptics meeting an opponent at each moment is given by $S * (E/N)$. Symmetrically, the number of environmentalists meeting a skeptic is given by $E * (S/N)$. Of course, $E * (S/N) = S * (E/N)$.

During a bilateral meeting, the degree of persuasiveness of an environmentalist to convince a skeptic is given by $\beta_1 \geq 0$, while $\beta_2 \geq 0$ represents the degree of persuasiveness of a skeptic to convince an environmentalist.

Furthermore, we suppose that both the types can also change their opinion spontaneously. We denote by $\gamma_1 \geq 0$ the share of skeptics who, spontaneously, become environmentalists and, by $\gamma_2 \geq 0$, the share of environmentalists who, spontaneously, become skeptics.

Proposition 2 *Opinion dynamics are given by*

$$\dot{s} = s(1-s)\left(\beta_2 - \beta_1 + \frac{\gamma_2}{s} - \frac{\gamma_1}{1-s}\right). \tag{28}$$

Proof. See the Appendix. ■

If $\beta_2 > \beta_1$, skeptics' persuasion force is stronger than the environmentalists' one. Conversely, if $\beta_1 > \beta_2$, the environmentalists' persuasion force is dominant.

Environmental quality influences the spread of climate skepticism and, then, β_i and γ_i (with $i = 1, 2$) can be represented as functions of the pollution level P. Specifically, a higher pollution level proves that economic activities are harmful to the environment. On the one hand, this reduces the persuasiveness of skeptics and increases that of environmentalists (respectively, $\beta_2'(P) < 0$ and $\beta_1'(P) > 0$). On the other hand, the share of skeptics who, spontaneously, become environmentalists, increases ($\gamma_1'(P) > 0$), and the share of environmentalists who, spontaneously, become skeptics, decreases ($\gamma_2'(P) < 0$).

Assumption 3 *For any $P \geq 0$, $\beta_2'(P) < 0 < \beta_1'(P)$ with*

$$\lim_{P\to 0} \beta_1(P) = 0 \text{ and } \lim_{P\to\infty} \beta_1(P) = \infty, \tag{29}$$

$$\lim_{P\to 0} \beta_2(P) = \infty \text{ and } \lim_{P\to\infty} \beta_2(P) = 0, \tag{30}$$

and $\gamma_2'(P) < 0 < \gamma_1'(P)$ with

$$\lim_{P\to 0} \gamma_1(P) = 0 \text{ and } \lim_{P\to\infty} \gamma_1(P) = \infty, \tag{31}$$

$$\lim_{P\to 0} \gamma_2(P) = \infty \text{ and } \lim_{P\to\infty} \gamma_2(P) = 0. \tag{32}$$

Let us introduce the political elasticities:

$$\varepsilon_1(P) \equiv \frac{P\beta_1'(P)}{\beta_1(P)} > 0 \text{ and } \varepsilon_2(P) \equiv \frac{P\beta_2'(P)}{\beta_2(P)} < 0, \tag{33}$$

$$\eta_1(P) \equiv \frac{P\gamma_1'(P)}{\gamma_1(P)} > 0 \text{ and } \eta_2(P) \equiv \frac{P\gamma_2'(P)}{\gamma_2(P)} < 0. \tag{34}$$

In addition, during the *gilets-jaunes* crisis that began in France in November 2018, the French government was led to abandon the increase in the carbon tax on fuel. To account for these pressures from skeptics on green fiscal policy, we assume that the carbon-tax rate depends on the degree of environmental skepticism: $\tau'(s) < 0$. This is what we call the Yellow Vests Effect (YVE). The green policy of a non-populist government is limited by the rise of a populist party or, when a populist party comes to power, it reduces or abolishes green taxes. The election of the second Trump is a recent example of this. Indeed, in the first 100 days of his new presidency, his administration scrapped the majority of Biden's green policies and favored the fossil fuel industry. The Guardian reports that 145 measures were taken in this short period to remove rules protecting the environment in the US.[8] $\tau(s)$ must verify certain properties summarized by the following assumption.

Assumption 4 $\tau'(s) < 0$ *with* $\tau(0) = \bar{\tau} \leq 1$ *and* $\tau(1) = 0$.

We introduce the fiscal elasticity:

$$\varepsilon_\tau(s) \equiv \frac{s\tau'(s)}{\tau(s)} < 0. \tag{35}$$

to capture the sensitivity of green fiscal policy to environmental skepticism.

For instance, Assumption 4 is satisfied by the explicit function τ:

$$\tau(s) = \bar{\tau}(1 - s^\pi), \tag{36}$$

with $\pi > 0$. We observe that $\tau(0) = \bar{\tau} \leq 1$, $\tau(1) = 0$ and

$$\varepsilon_\tau(s) = -\pi \frac{s^\pi}{1 - s^\pi}. \tag{37}$$

In particular, when $\pi = 1$, we obtain

$$\tau(s) = \bar{\tau}(1 - s), \tag{38}$$

$$\varepsilon_\tau(s) = -\frac{s}{1 - s}. \tag{39}$$

3 Equilibrium

The economy is made up of three markets: the labor market, the capital market and the goods market. In general equilibrium, these markets clear together.[9]

[8] See the website: https://www.theguardian.com/environment/2025/may/01/trump-air-climate-pollution-regulation-100-days

[9] For a formal treatment, see the proof of Proposition 3.

Proposition 3 *The dynamic general equilibrium is a represented by the following system:*

$$\dot{\mu} = (\delta + \theta - [1 - \tau(s)]\rho(k))\mu, \tag{40}$$

$$\dot{k} = [1 - \tau(s)] f(k) - \delta k - c(\mu, P), \tag{41}$$

$$\dot{P} = -aP + [b - d\tau(s)] f(k), \tag{42}$$

$$\dot{s} = s(1 - s)\left[\beta_2(P) - \beta_1(P) + \frac{\gamma_2(P)}{s} - \frac{\gamma_1(P)}{1 - s}\right]. \tag{43}$$

Proof. See the Appendix. ■

The system results from the addition of three blocks: economic, ecological and political. More precisely, equations (40) and (41) represent the Ramsey model, augmented by the pollution process (42) and the opinion dynamics (43). μ is a jump variable while k, P and s are three predetermined variables.[10]

3.1 Steady state

Let us suppose that production has a larger impact on pollution than depollution.

Assumption 5 $b \geq d$.

Assumption 5 ensures the existence of a non-negative pollution stock at the steady state.

Proposition 4 *Under Assumptions 1 to 5, there exists at least a non-trivial steady state* (μ^*, k^*, P^*, s^*). *This steady state is unique.*

Capital k^* *and pollution* P^* *depend on populism* s^**:*

$$k^* = \rho^{-1}\left(\frac{\delta + \theta}{1 - \tau(s^*)}\right) \equiv k(s^*) > 0, \tag{44}$$

$$P^* = \frac{b - d\tau(s^*)}{a} f(k(s^*)) \equiv P(s^*) > 0, \tag{45}$$

where s^* *is the unique solution to*

$$\beta_2(P(s)) - \beta_1(P(s)) = \frac{\gamma_1(P(s))}{1 - s} - \frac{\gamma_2(P(s))}{s}. \tag{46}$$

The consumption demand and its marginal utility are given by

$$c^* = [1 - \tau(s^*)] f(k^*) - \delta k^* > 0, \tag{47}$$

$$\mu^* = u_c(c^*, P^*) > 0. \tag{48}$$

Proof. See the Appendix. ■

In the isoelastic case (22), only μ^* depends on (ε, η), while k^*, P^*, s^* and c^* don't.

[10] According to (17), for any given pollution level P, the choice of the control variable c gives the value of the shadow price of capital μ.

Interestingly, k^* does not depend on the level of pollution. This is because pollution is a pure externality for the representative household.[11]

3.2 Comparative statics

In this section, we consider the functional form (36) for the tax rate $\tau(s)$ and we focus on the impact of YVE, that is $\bar{\tau}$, on the steady state. Let us define the steady state as a function of YVE: $s^* = s^*(\bar{\tau})$, $k^* = k^*(\bar{\tau})$, $P^* = P^*(\bar{\tau})$.

Proposition 5 *Under the functional form (36), the impacts of YVE $\bar{\tau}$ on s^*, k^* and P^* are the following:*

$$\frac{\bar{\tau}}{s^*}\frac{ds^*}{d\bar{\tau}} > 0,\ \frac{\bar{\tau}}{k^*}\frac{dk^*}{d\bar{\tau}} < 0 \text{ and } \frac{\bar{\tau}}{P^*}\frac{dP^*}{d\bar{\tau}} < 0. \tag{49}$$

Proof. See the Appendix. ■

Proposition 5 deserves an economic interpretation. Since the tax is levied on the level of production, a higher rate provides an incentive to reduce production. This implies a lower demand for inputs, which translates into a lower level of capital in the long run. In addition, the lower level of production leads to a reduction in pollutant emissions and therefore in the level of pollution in the long term. In line with Assumption 3, the decrease in pollution induced by a higher tax rate increases the persuasiveness of skeptics and decreases that of environmentalists: the share of skeptics increases in the long term.

4 Opinion cycles

Since the seminal Heal's (1982) contribution, it is known that a limit cycle can arise near the steady state of a Ramsey economy when a pollution externality increases the marginal utility of consumption (compensation effect). Our goal now is to understand how robust this result is under populism. More precisely, we propose to study the dynamics around the non-trivial steady state, focusing on the conditions under which a limit cycle can occur (Hopf bifurcation). This will allow us to understand the effect of populism on macroeconomic stability.

In the following, for simplicity, we consider the utility function (22) to represent the households' preferences, and a Cobb-Douglas production function (with unit elasticity of factor substitution: $\sigma = 1$), that is $f(k) = Ak^\alpha$. Moreover, we assume a linear tax function with $\bar{\tau} = 1$ (i.e., $\tau(s) = 1 - s$). Finally, we introduce isoelastic political functions:

$$\beta_1(P) = B_1 P^{\varepsilon_1} \text{ with } \varepsilon_1 > 0, \tag{50}$$

$$\beta_2(P) = B_2 P^{\varepsilon_2} \text{ with } \varepsilon_2 < 0, \tag{51}$$

$$\gamma_1(P) = C_1 P^{\eta_1} \text{ with } \eta_1 > 0, \tag{52}$$

$$\gamma_2(P) = C_2 P^{\eta_2} \text{ with } \eta_2 < 0. \tag{53}$$

[11] While this paper focuses on the market solution, other contributions have considered instead the planner's solution in economies with no political contagion. The interested reader is referred to Van der Ploeg and Withagen (1991).

Proposition 6 *The equilibrium is locally unique.*

Proof. See the Appendix. ■

Proposition 6 rules out local indeterminacy and fluctuations due to self-fulfilling expectations.[12] This implies that, given the initial values of the state variables (namely $k(0)$, $P(0)$, $s(0)$), there exists a unique value of the control variable, here $c(0)$, allowing convergence to the unique non-trivial steady state (k^*, P^*, s^*, c^*). Uniqueness is a robust property of equilibrium in our context.[13]

A suitable parameter to study the occurrence of limit cycles is η. Indeed, η captures the degree of pollution externality and, since $\varepsilon_{cP} = \eta(\varepsilon - 1)$, it amplifies the compensation effect or the distaste effect, that is the forces at work responsible for the fluctuations.

The following proposition involves the critical expression D_2^H whose terms (A_i, Σ_j, T and D) depend on the fundamental parameters of the model. To lighten the exposition, their cumbersome expressions are relegated and detailed in the Appendix (see Lemma 8).

Proposition 7 *A limit cycle arises through a Hopf bifurcation around the non-trivial steady state when η crosses the critical value, that is*

(1) if $a < \theta$, when

$$\eta = \eta_H \equiv \frac{\varepsilon}{1-\varepsilon}\frac{D_2^H}{a}\frac{b - d(1-s)}{s}\frac{\theta+\delta}{\theta+\delta(1-\alpha)}, \tag{54}$$

where

$$D_2^H \equiv \frac{T\Sigma_3 + TO_2\Sigma_2 - 2O_2\Sigma_3 - T\sqrt{(O_2\Sigma_2 - \Sigma_3)^2 + 4DO_2(\theta - a)}}{2\alpha A_1 A_2 O_2(\theta - a)}; \tag{55}$$

(2) if $a > \theta$ and $C_1 = C_2 = 0$ (that is, without spontaneous changes in mind: $\gamma_1(P) = \gamma_2(P) = 0$ for any P), when

$$\eta = \tilde{\eta}_H \equiv \frac{\varepsilon}{1-\varepsilon}\frac{1}{a[\theta + (1-\alpha)\delta]}\left[\Sigma_2 - \left(\frac{\Sigma_3}{T} + D\frac{T}{\Sigma_3}\right)\right], \tag{56}$$

provided that $\Sigma_3 < 0$.

Proof. See the Appendix. ■

To interpret Proposition 7, we need to consider separately the cases $\varepsilon \lesseqgtr 1$ and, then, take into account the corresponding restrictions to ensure that $\eta_H > 0$ and $\tilde{\eta}_H > 0$. The expression of η_H is cumbersome and computations

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[12] Since Azariadis (1981), it is known that local indeterminacy implies the existence of self-fulfilling prophecies. These important phenomena, often at the root of financial and economic crises, occur when agents expect a change and adapt their behavior so that their expectations actually come true.

[13] Interestingly, it is possible to show that local determinacy holds with more general isoelastic fundamentals (constant elasticities of factor substitution and intertemporal substitution) and the power taxation function (36).

are far from being easy. However, according to the existing literature on the cross effects in preferences without opinion dynamics, the existence of limit cycles through a Hopf bifurcation requires a compensation effect ($\varepsilon > 1$) under sufficient high degree of pollution inertia ($a < \theta$). Conversely there is no room for Hopf bifurcations under a distaste effect ($\varepsilon < 1$), whatever the degree of pollution inertia.[14] Our model generalizes this basic framework with opinion dynamics and, unsurprisingly, we recover the occurrence of limit cycles under a compensation effect. What is new is the possibility of limit cycles under the distaste effect because of the opinion dynamics.

Focus first on the case of compensation effect to understand how populism makes cycles more likely. Assume an exogenous increase in the level of pollution today. As a result of the compensation effect, the household increases its current consumption and reduces its savings, thereby reducing tomorrow's capital stock. Less capital also means less production and, therefore, a lower level of pollution, and so on. As seen above, this explanation is standard in Ramsey economies with pollution.[15]

However, populism makes these cycles more likely. Indeed, the more pollution there is, the more convincing the environmentalists are, which reduces the proportion of skeptics in the population. The pressure for an environmental policy becomes higher and the green tax increases, reducing the level of pollution. As before, an increase in pollution is followed by a decrease in pollution: the two mechanisms, consumption and populism, move in the same direction.

Focus now on the case of distaste effect. The previous mechanism suggests that a green taxation highly sensitive to populism can promote the occurrence of cycles even in the unfavorable case of distaste. As seen above, because of the complicated expressions of η_H, it is not possible to prove analytically this conjecture. However, it is numerically. More precisely, we will show in the next section that there is room for stable cycles under distaste effect ($\varepsilon < 1$) if the rate of pollution absorption is sufficiently high ($a > \theta$). In this respect, we can affirm that populism exacerbates economic volatility in a polluted world.

5 Simulations

In the previous sections, we have provided the necessary and sufficient generic conditions for the occurrence of limit cycles through a Hopf bifurcation around the non trivial-steady state. However, our analysis has not permitted to know whether the occurrence of cycles requires a distaste ($\varepsilon < 1$) or a compensation effect ($\varepsilon > 1$). In the current section, we show numerically that both these effects can lead to a Hopf bifurcation.

This is an important and new result because, it is known, there is no room for Hopf bifurcation under a distaste effect with no opinion dynamics. Simulations give also us the opportunity to study the stability of the limit cycle arising through the Hopf bifurcation, that is its supercriticity. Importantly, using the

[14] The reader is referred to Propositions 11 and 12 in Bosi and Desmarchelier (2018).

[15] See Bosi and Desmarchelier (2018) among others.

Matcont package for Matlab, we are able to simulate directly the original nonlinear system (40)-(43) instead of a linear approximation. These simulations are fully consistent with our analytical results, obtained through the (Jacobian) linearization.

As in the previous section, we focus on the explicit functional forms (22), (50), (51), (52), (53) and a linear tax rate: $\tau(s) = 1 - s$.

5.1 Case $a < \theta$ and $\varepsilon > 1$

To simplify the simulation, we normalize the political and fiscal parameters:

Parameter	ε_1	ε_2	η_1	η_2	B_1	B_2	C_1	C_2
Value	1	-1	1	-1	1	1	1	1

(57)

Under calibration (57), we get $\beta_1(P) = \gamma_1(P) = P$ and $\beta_2(P) = \gamma_2(P) = 1/P$.

The non-trivial steady state becomes:

$$k^* = \left(\frac{\alpha A s^*}{\delta + \theta}\right)^{\frac{1}{1-\alpha}}, \tag{58}$$

$$P^* = P(s^*) = A\frac{b - d(1 - s^*)}{a}\left(\frac{\alpha A s^*}{\delta + \theta}\right)^{\frac{\alpha}{1-\alpha}}, \tag{59}$$

$$\mu^* = \left(\frac{P^* \eta^{\frac{\varepsilon - 1}{\varepsilon}}}{s^* A k^{*\alpha} - \delta k^*}\right)^{\varepsilon}, \tag{60}$$

where $s^* \in (0,1)$ is solution to (46), that is to

$$P(s)^2 = \frac{1 - s^2}{s(2 - s)}. \tag{61}$$

The traditional case of a strong pollution inertia ($a < \theta$) under a compensation effect ($\varepsilon > 1$) is known to generate a Hopf bifurcation.[16] As expected, this case also leads to a Hopf bifurcation in our more general setup with populism.

We complete the calibration (57) as follows:

Parameter	θ	a	A	δ	α	ε	b	d
Value	0.01	0.005	1	0.025	1/2	4	0.001	0.0005

(62)

$\alpha = 1/2$ simplifies the computation of the roots of equation (61). We observe also that $\varepsilon = 4$ implies $\varepsilon_{cP} = \eta(\varepsilon - 1) > 0$ (compensation effect).

Calibrations (57) and (62) lead to the following steady state value:

$$(k^*, P^*, s^*) \approx (47.594, 1.023, 0.48292). \tag{63}$$

[16] See Bosi and Desmarchelier (2018) among others.

Using (54), we compute the critical value:

$$\eta_H = \frac{\varepsilon}{1-\varepsilon} \frac{D_2^H}{a} \frac{b - d\,(1-s^*)}{s^*} \frac{\delta+\theta}{\theta+\delta\,(1-\alpha)} \approx 13.693, \tag{64}$$

where $D_2^H \approx -21.5$ is given by (55). Finally, using $\eta = \eta_H \approx 13.693$, we can also compute the stationary multiplier:

$$\mu^* = \left(\frac{P^* \eta^{\frac{\varepsilon-1}{\varepsilon}}}{s^* A k^{*\alpha} - \delta k^*}\right)^{\varepsilon} \approx 0.12095. \tag{65}$$

We implement the dynamic system (40)-(43) in Matcont. The software finds independently a Hopf bifurcation at $\eta_H \approx 13.693091$. When $\eta = \eta^H$ the real eigenvalues are given by:

$$(\lambda_1, \lambda_2) \approx (-1.99968, 0.00260269)\,, \tag{66}$$

while the non-real (purely imaginary) eigenvalues by

$$(\lambda_3, \lambda_4) \approx (-0.0340141, 0.0340141)\, i. \tag{67}$$

The corresponding first Lyapunov coefficient evaluated with Matcont is negative: $l_1 \approx -3.406572 * 10^{-6}$, meaning that the Hopf bifurcation is supercritical.

Figure 1 proposes a projection of the stable limit cycle arising around the non-trivial steady state in the (μ, P) space as well as the vector field around it.

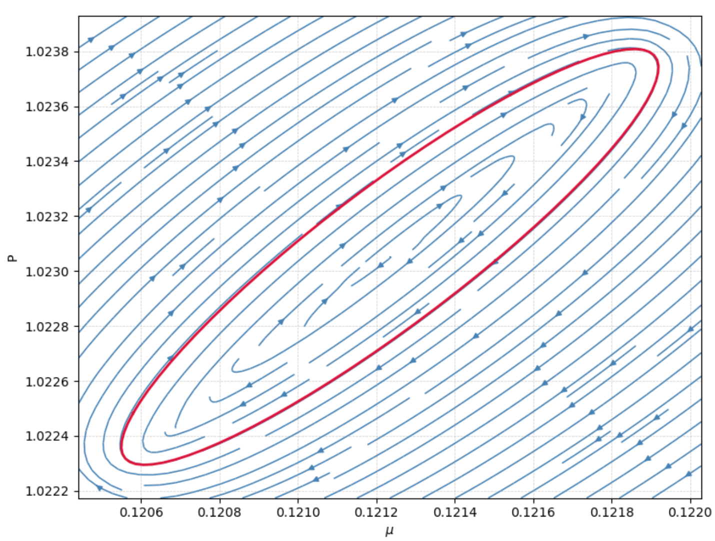

Figure 1: Stable limit cycle

5.2 Case $a > \theta$ and $\varepsilon < 1$

We know that limit cycles arise through a Hopf bifurcation in the basic model without opinion cycles if $\varepsilon > 1$ (compensation effect) *and* $a < \theta$ (pollution inertia).[17] Conversely, if $\varepsilon < 1$ (distaste effect) *or* $a > \theta$ (fast pollution absorption), limit cycles are impossible.

The introduction of political dynamics promotes the emergence of limit cycles even when they fail to exist in the basic model without political contagion. To illustrate how powerful opinion waves are, we show the possibility of cycles in the less favorable case: $\varepsilon < 1$ *and* $a > \theta$. A numerical exercise in the simplified version of the model with no spontaneous change in mind, corresponding to case (2) in Proposition 7, is enough to highlight this possibility. More precisely, reconsider calibration (57) with, now, $C_1 = C_2 = 0$. Interestingly, equation (46) reduces to

$$P = \frac{1}{P} \tag{68}$$

leading to a simple steady state: $P = 1$. To simplify more, we also assume that $\alpha = 1/2$ and $b = d$. The unique non-trivial steady state becomes:

$$s^* = \frac{1}{A}\sqrt{\frac{2a\,(\delta + \theta)}{d}}, \tag{69}$$

$$k^* = \left[\frac{s^* A}{2\,(\delta + \theta)}\right]^2, \tag{70}$$

$$P^* = 1, \tag{71}$$

$$\mu^* = \left(s^* A k^{*\frac{1}{2}} - \delta k^*\right)^{-\varepsilon}. \tag{72}$$

We complete the calibration as follows:

Parameter	θ	a	A	δ	α	ε	b	d
Value	0.005	0.01	1	0.025	1/2	6/100	0.001	0.001

(73)

The steady state becomes:

$$(\mu^*, k^*, P^*, s^*) \approx (0.89959, 166.67, 1, 0.77460)\,. \tag{74}$$

Using (56), we obtain the Hopf critical value $\tilde{\eta}_H = 2.3051$ and, interestingly,

$$\Sigma_3 = -2.5202 \times 10^{-5} < 0$$

(see Proposition 7).

The real eigenvalues corresponding to this Hopf bifurcation value are given by

$$(\lambda_1, \lambda_2) \approx (-0.12763, 0.12263)\,, \tag{75}$$

[17] See Propositions 11 and 12 in Bosi and Desmarchelier (2018).

while the non-real (purely imaginary) eigenvalues are given by

$$(\lambda_3, \lambda_4) \approx (-0.070995, 0.070995)\, i. \quad (76)$$

The corresponding first Lyapunov coefficient evaluated with Matcont is negative: $l_1 \approx -1.668469 * 10^{-6} < 0$, meaning that the Hopf bifurcation is supercritical.

Figure 2 proposes a projection of the stable limit cycle arising around the non-trivial steady state in the (s, P) space as well as the vector field around it.

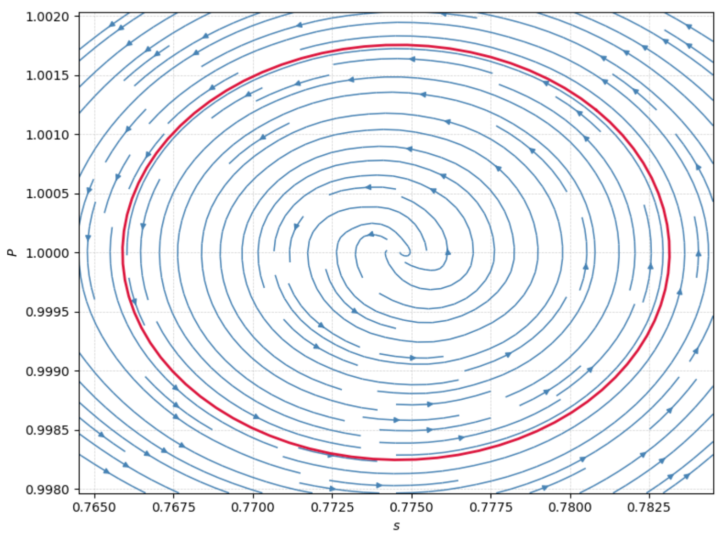

Figure 2: Stable limit cycle

5.3 Local determinacy

Simulations help us grasp the deeper meaning of Proposition 6. Let us explain why the equilibrium remains unique before and after the bifurcation by focusing on the Hopf bifurcations in our simulations. They are supercritical, meaning the cycles are stable.

There are two real eigenvalues: one remains negative (stable) during the bifurcation, the other positive (unstable). There are also two non-real conjugate eigenvalues, whose real part goes through zero.

Before the bifurcation, three eigenvalues are stable; afterward, three eigenvalues become unstable. Note that the center subspace is two-dimensional.

The non-predetermined variable (μ) can adjust to neutralize the unstable (positive) eigenvalue.

Before the bifurcation, the stable manifold is three-dimensional. The three predetermined variables (k, P and s) fix a point in this manifold, which is the starting point of the equilibrium trajectory that lies there and converges to the steady state along a three-dimensional spiral. The trajectory vaguely resembles a converging conical spiral in the (k, P, s)-subspace.

When the real part becomes positive while remaining close to zero, the stable manifold becomes one-dimensional and the starting point given by the three predetermined variables is close to but outside this manifold. The equilibrium trajectory then diverges from the steady state to converge towards the surrounding stable cycle by a three-dimensional spiral. This trajectory roughly resembles a diverging conical spiral (see Figures 1 and 2).

In both cases (one-dimensional or three-dimensional stable manifold), the equilibrium trajectory is unique because its starting point is uniquely fixed by the three state variables.

5.4 Discussion

The analytical conclusions of Proposition (7) underline the possibility of limit cycles through a Hopf bifurcation around the non-trivial steady state of the economy. This proposition shows that cycles can appear whatever the effect of pollution on the marginal utility of consumption (either compensation with $\varepsilon > 1$ or distaste with $\varepsilon < 1$), and whatever the inertia of pollution ($a \lesseqgtr \theta$). We have complemented and illustrated the analytical results with two numerical simulations.

It was not surprising that limit cycles arise under a compensation effect, as we know that they appear in a particular case of ours, the very basic Ramsey model with pollution, under the joint action of a compensation effect and pollution inertia ($\varepsilon > 1$ and $a < \theta$).[18] So, a fortiori, the economy can experience a Hopf bifurcation in our more general framework with political contagion.

Our second simulation focuses on the case of distaste ($\varepsilon < 1$) and is much more intriguing. Indeed, this case is known to rule out the occurrence of Hopf bifurcations in the basic model with pollution, but without political contagion. However, when contagion is taken into account, limit cycles arise in the case of rapid pollution absorption ($a > \theta$): importantly, political contagion appears as a destabilizing force in the economy.

Beyond the purely theoretical interest of these results, our model also delivers three policy recommendations.

(1) As mentioned in the introduction, populism places great importance on economic rather than environmental issues (Lockwood, 2018). If a populist government cares about macroeconomic stability, which seems to be the case, its program must implement green policies rather than reject them. Populists concerned about unpleasant macroeconomic volatility need to be more environmentally oriented rather than skeptical. This argument goes hand in hand with the vulnerability of the poor to climate change and points out that governments, even populist ones, has to adapt fiscal policies to environmental objectives rather than to adapt them to please a particular community.[19]

[18] See Bosi and Desmarchelier (2018).
[19] That is, to consider $\tau \equiv \tau(P)$ rather than $\tau \equiv \tau(s)$.

(2) Interestingly, political contagion only affects the economy through the YVE, so the elasticity of the tax rate with respect to environmental skepticism plays a key role. Its value depends on the ability of skeptics to mobilize and coordinate to put pressure on the government. As Mutascu et al. (2025) point out, online social media such as Facebook, Instagram or Twitter (now X) play a major role in explaining the spread of right-wing populism and, hence, climate skepticism (Lockwood, 2018). Thus, the massive use of social media by the population will probably tend to increase the scale of YVE in the future and promote instability. Our work recommends that a government concerned about macroeconomic volatility regulate social media, for instance by limiting the spread of anti-science.

(3) Finally, the existence of limit cycles raises the question of sustainability. The concept of sustainable development was popularized by the World Commission on Environment and Development in its famous Brundtland Report (Our Common Future, 1987) as "development that meets the needs of the present without compromising the ability of future generations to meet their own needs." This definition can be translated into economics in terms of intertemporal utility function.

Indeed, following Pezzey (1992) or Arrow et al. (2012) among others, a trajectory is sustainable if utility does not decrease along it. When the economy undergoes a limit cycle, agents are confronted with utility fluctuations that violate the sustainability criterion by causing intergenerational inequity. This is a last but not least argument in favor of combating climate skepticism and populism.

6 Conclusion

Our article is a first attempt to consider populism and pollution together in a dynamic general equilibrium model. We adapt a SIS model to represent the spread of climate skepticism and populism in society, dividing the population into two mutually influencing groups: climate skeptics and environmentalists.

Since right-wing populism is known for prioritizing economic over environmental issues (Lockwood, 2018), we equate populists with skeptics. This political bloc is integrated in a Ramsey model where pollution comes from production. Environmental policy consists of a green tax levied on production to finances depollution according to a balanced-budget rule. To take into account the political pressure of populists, we assume that the ecotax rate decreases in the share of skeptics in the population.

Our analysis, at the crossroads of economic, political and environmental sciences and epidemiology, reveals that populism promotes the emergence of stable limit cycles around the steady state through a Hopf bifurcation, regardless of the effects of pollution on consumption demand.

Interestingly, in the absence of populism, a Hopf bifurcation only appears in a Ramsey model when pollution increases the marginal utility of consumption (compensation effect). In other words, populism exacerbates pollution-induced volatility. In this regard, even if right-wing parties place a high priority on short-term economic performance, to manage macroeconomic volatility, they would be better off considering environmental policies rather than rejecting them *a priori*.

Declarations

Data Availability: Not applicable.
Ethics Approval: Not applicable.
Funding Declaration: Not applicable.

7 Appendix

Proof of Proposition 2

The opinion dynamics are given by:

$$\dot{E} = \beta_1 E \frac{S}{N} - \beta_2 S \frac{E}{N} + \gamma_1 S - \gamma_2 E, \tag{77}$$

$$\dot{S} = \beta_2 S \frac{E}{N} - \beta_1 E \frac{S}{N} + \gamma_2 E - \gamma_1 S. \tag{78}$$

Since $\dot{E} + \dot{S} = \dot{N} = 0$, (77) and (78) are equivalent:

$$\dot{E}+\dot{S} = \left(\beta_1 E \frac{S}{N} - \beta_2 S \frac{E}{N} + \gamma_1 S - \gamma_2 E\right) + \left(\beta_2 S \frac{E}{N} - \beta_1 E \frac{S}{N} + \gamma_2 E - \gamma_1 S\right) = 0. \tag{79}$$

According to (78), $\dot{s}/s = \dot{S}/S - \dot{N}/N = \dot{S}/S$ implies

$$\frac{\dot{s}}{s} = (\beta_2 - \beta_1) \frac{E}{N} + \gamma_2 \frac{E}{S} - \gamma_1 = (\beta_2 - \beta_1)(1 - s) + \gamma_2 \frac{1-s}{s} - \gamma_1, \tag{80}$$

and, finally, (28). ■

Proof of Proposition 3

Equilibrium in the labor market means: $L = Nl = N$ since $l = 1$; while, in the capital market: $K = Nh = Lh$, that is $k = h$. In the good market, the aggregate demand is also equal to the aggregate supply: $C + \left(\dot{K} + \delta K\right) + G = Y$, that is

$$c + \left(\frac{\dot{H}}{N} + \delta h\right) + \tau y = c + \left(\dot{h} + \delta h\right) + \tau f(k) = y = f(k). \tag{81}$$

We get $c + \left(\dot{h} + \delta h\right) = (1 - \tau) f(k)$ or, equivalently,

$$\dot{k} = (1 - \tau) f(k) - \delta k - c. \tag{82}$$

Putting together (1), (2), (15), (19), (26), (27), (82), under Assumptions 3 and 4, we obtain system (40)-(43). Notice that, in equilibrium, the household's budget constraint (16) corresponds to the goods market clearing:

$$\dot{h} = (r-\delta)\, h + w - c = [(1-\tau)\, f'(k) - \delta]\, k + (1-\tau)\, [f(k) - kf'(k)] - c, \quad (83)$$

that is to (82). ■

Proof of Proposition 4

At the steady state, according to equation (40), we have

$$\rho(k^*) = (\delta+\theta) / [1-\tau(s^*)], \quad (84)$$

that is (44).

Under Assumption 1 and 4:

$$\lim_{s\to 0} \frac{\delta+\theta}{1-\tau(s)} = \infty = \lim_{k\to 0} \rho(k) \text{ and } \lim_{s\to 1} \frac{\delta+\theta}{1-\tau(s)} = \delta+\theta. \quad (85)$$

We obtain $\lim_{s\to 0} k(s) = 0$ and $\lim_{s\to 1} k(s) = k_R$, where $k_R \equiv \rho^{-1}(\delta+\theta)$ is the Modified Golden Rule of the basic Ramsey model.

Focus now on equation (42). At the steady state, we get (45). We observe that $P'(s) > 0$. Moreover, $\lim_{s\to 0} P(s) = 0$. We find also

$$\lim_{s\to 1} P(s) = \frac{b}{a} f(k_R) = \frac{b}{a} f\left(\rho^{-1}(\delta+\theta)\right) \equiv \bar{P} > 0. \quad (86)$$

Consider equation (43). At the steady state,

$$\varphi(s) \equiv (1-s)\left(s\left[\beta_2(P(s)) - \beta_1(P(s))\right] + \gamma_2(P(s))\right) - s\gamma_1(P(s)) = 0. \quad (87)$$

Under Assumption 3,

$$\lim_{s\to 0} \varphi(s) = \lim_{s\to 0} \left(s\left[\beta_2(P(s)) - \beta_1(P(s))\right] + \gamma_2(P(s))\right) = +\infty, \quad (88)$$

$$\lim_{s\to 1} \varphi(s) = -s\gamma_1\left(\bar{P}\right) < 0. \quad (89)$$

Since $\varphi(s)$ is continuous, there exists at least one $s \in (0,1)$ such that $\varphi(s) = 0$.

Let s^* be a steady state. According to (44) and (45), we obtain the corresponding values for capital intensity and pollution level: $k^* \equiv k(s^*)$ and $P^* \equiv P(s^*)$. Moreover, (41) entails (47) and, finally, (17) yields (48).

Focus now on uniqueness.

Equation (87) becomes (46). We know that $P'(s) > 0$. Then the LHS decreases, while the RHS increases. In addition, under Assumption 3,

$$\lim_{s\to 0} \left[\beta_2(P(s)) - \beta_1(P(s))\right] = \infty, \quad (90)$$

$$\lim_{s\to 1} \left[\beta_2(P(s)) - \beta_1(P(s))\right] = \beta_2\left(\bar{P}\right) - \beta_1\left(\bar{P}\right), \quad (91)$$

$$\lim_{s\to 0} \left[\frac{\gamma_1(P(s))}{1-s} - \frac{\gamma_2(P(s))}{s}\right] = -\infty, \quad (92)$$

$$\lim_{s\to 1} \left[\frac{\gamma_1(P(s))}{1-s} - \frac{\gamma_2(P(s))}{s}\right] = \infty, \quad (93)$$

with $\bar{P} \equiv f\left(\rho^{-1}(\delta+\theta)\right) b/a$. Therefore a unique non-trivial steady state exists.
■

Proof of Proposition 5

We introduce useful blocks, which are positive under Assumption 5:

$$S_0 \equiv \frac{1-s^\pi}{s^\pi} > 0, \tag{94}$$

$$S_1 \equiv \frac{1}{\bar{\tau} s^\pi} - \frac{1-s^\pi}{s^\pi} = \frac{1-\tau(s)}{\bar{\tau} s^\pi} > 0, \tag{95}$$

$$S_2 \equiv \frac{b}{d}\frac{1}{\bar{\tau} s^\pi} - \frac{1-s^\pi}{s^\pi} > 0, \tag{96}$$

$$Q_1 \equiv \frac{s}{1-s}\frac{\gamma_1(P)}{1-s} + \frac{\gamma_2(P)}{s} > 0, \tag{97}$$

$$Q_2 \equiv \beta_1(P)\varepsilon_1(P) - \beta_2(P)\varepsilon_2(P) + \eta_1(P)\frac{\gamma_1(P)}{1-s} - \eta_2(P)\frac{\gamma_2(P)}{s} > 0. \tag{98}$$

Consider $\tau(s) \equiv \bar{\tau}(1-s^\pi)$. The steady state (s,k,P) is solution to system:

$$[1-\bar{\tau}(1-s^\pi)]\rho(k) = \delta+\theta, \tag{99}$$

$$[b-d\bar{\tau}(1-s^\pi)] f(k) = aP, \tag{100}$$

$$\beta_2(P) - \beta_1(P) = \frac{\gamma_1(P)}{1-s} - \frac{\gamma_2(P)}{s}. \tag{101}$$

Totally differentiating with respect to $(s,k,P,\bar{\tau})$, we obtain

$$\pi\frac{ds}{s} + S_1\frac{k\rho'(k)}{\rho(k)}\frac{dk}{k} = S_0\frac{d\bar{\tau}}{\bar{\tau}}, \tag{102}$$

$$\pi\frac{ds}{s} + S_2\frac{kf'(k)}{f(k)}\frac{dk}{k} - S_2\frac{dP}{P} = S_0\frac{d\bar{\tau}}{\bar{\tau}}, \tag{103}$$

$$Q_1\frac{ds}{s} + Q_2\frac{dP}{P} = 0, \tag{104}$$

and, replacing (4) and (7),

$$\begin{bmatrix} \pi & -\frac{1-\alpha(k)}{\sigma(k)}S_1 & 0 \\ \pi & \alpha(k)S_2 & -S_2 \\ Q_1 & 0 & Q_2 \end{bmatrix} \begin{bmatrix} \frac{\bar{\tau}}{s}\frac{ds}{d\bar{\tau}} \\ \frac{\bar{\tau}}{k}\frac{dk}{d\bar{\tau}} \\ \frac{\bar{\tau}}{P}\frac{dP}{d\bar{\tau}} \end{bmatrix} = \begin{bmatrix} S_0 \\ S_0 \\ 0 \end{bmatrix}. \tag{105}$$

Solving the system, we find the elasticities:

$$\frac{\bar{\tau}}{s^*}\frac{ds^*}{d\bar{\tau}} = \frac{S_0 Q_2\left[\alpha(k)S_2 + \frac{1-\alpha(k)}{\sigma(k)}S_1\right]}{\alpha(k)\pi Q_2 S_2 + \frac{1-\alpha(k)}{\sigma(k)}S_1(\pi Q_2 + Q_1 S_2)} > 0, \tag{106}$$

$$\frac{\bar{\tau}}{k^*}\frac{dk^*}{d\bar{\tau}} = -\frac{Q_1 S_0 S_2}{\alpha(k)\pi Q_2 S_2 + \frac{1-\alpha(k)}{\sigma(k)}S_1(\pi Q_2 + Q_1 S_2)} < 0, \tag{107}$$

$$\frac{\bar{\tau}}{P^*}\frac{dP^*}{d\bar{\tau}} = -\frac{Q_1 S_0\left[\alpha(k)S_2 + \frac{1-\alpha(k)}{\sigma(k)}S_1\right]}{\alpha(k)\pi Q_2 S_2 + \frac{1-\alpha(k)}{\sigma(k)}S_1(\pi Q_2 + Q_1 S_2)} < 0. \tag{108}$$

■

Lemma 8 *The sums of the principal minors of the Jacobian matrix of system (40)-(43) are given by:*

$$S_1 = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = \theta - a + O_2, \tag{109}$$
$$S_2 = \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4 = \Sigma_2 - \alpha A_1 A_2 D_2, \tag{110}$$
$$S_3 = \lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_4 + \lambda_1\lambda_3\lambda_4 + \lambda_2\lambda_3\lambda_4 = \Sigma_3 - \alpha A_1 A_2 D_2 O_2, \tag{111}$$
$$\begin{aligned} S_4 &= \lambda_1\lambda_2\lambda_3\lambda_4 \\ &= a\alpha A_1 D_1 \left(O_2 (1-\alpha) - B (1-s^*) \left[\alpha A_2 + d (1-\alpha)\right]\right), \end{aligned} \tag{112}$$

where $S_1 = T$ *is the trace and* $S_4 = D < 0$ *is the determinant, and*

$$A_1 \equiv \frac{\delta + \theta}{\alpha} > 0, \tag{113}$$
$$A_2 \equiv \frac{b - d(1-s)}{s} > 0, \tag{114}$$
$$D_1 \equiv \frac{A_1 - \delta}{\varepsilon} = \frac{1}{\varepsilon}\frac{c^*}{k^*} > 0, \tag{115}$$
$$D_2 \equiv a\eta (1-\varepsilon) \frac{D_1}{A_1 A_2} > 0 \Leftrightarrow \varepsilon < 1, \tag{116}$$
$$O_1 \equiv \varepsilon_2 \beta_2 (P^*) - \varepsilon_1 \beta_1 (P^*) + \eta_2 \frac{\gamma_2 (P^*)}{s^*} - \eta_1 \frac{\gamma_1 (P^*)}{1 - s^*} < 0, \tag{117}$$
$$O_2 \equiv -s^* \frac{\gamma_1 (P^*)}{1 - s^*} - (1 - s^*) \frac{\gamma_2 (P^*)}{s^*} \leq 0, \tag{118}$$
$$\Sigma_2 \equiv (\theta - a) O_2 + adB (1 - s^*) - a\theta - \alpha (1-\alpha) A_1 D_1, \tag{119}$$
$$\Sigma_3 \equiv a (1 - s^*) (d\theta - \alpha A_1 A_2) B - a\theta O_2 + \alpha (1 - \alpha) A_1 D_1 (a - O_2), \tag{120}$$

with

$$B \equiv -\frac{O_1}{A_2} > 0. \tag{121}$$

Proof of Lemma 8

The dynamic system (40)-(43) writes $\left(\dot{\mu}, \dot{k}, \dot{P}, \dot{s}\right)^T = f(\mu, k, P, s)$, where $f \equiv (f_1, f_2, f_3, f_4)^T$. The Jacobian matrix is given by

$$\begin{aligned} J &\equiv \begin{bmatrix} \frac{\partial f_1}{\partial \mu} & \frac{\partial f_1}{\partial k} & \frac{\partial f_1}{\partial P} & \frac{\partial f_1}{\partial s} \\ \frac{\partial f_2}{\partial \mu} & \frac{\partial f_2}{\partial k} & \frac{\partial f_2}{\partial P} & \frac{\partial f_2}{\partial s} \\ \frac{\partial f_3}{\partial \mu} & \frac{\partial f_3}{\partial k} & \frac{\partial f_3}{\partial P} & \frac{\partial f_3}{\partial s} \\ \frac{\partial f_4}{\partial \mu} & \frac{\partial f_4}{\partial k} & \frac{\partial f_4}{\partial P} & \frac{\partial f_4}{\partial s} \end{bmatrix} \\ &= \begin{bmatrix} 0 & \alpha (1-\alpha) A_1 \frac{\mu^*}{k^*} & 0 & -\alpha A_1 \frac{\mu^*}{s^*} \\ D_1 \frac{k^*}{\mu^*} & \theta & D_2 & A_1 \frac{k^*}{s^*} \\ 0 & \alpha A_1 A_2 & -a & d A_1 \frac{k^*}{s^*} \\ 0 & 0 & a (1 - s^*) \frac{O_1}{A_1 A_2} \frac{s^*}{k^*} & O_2 \end{bmatrix}, \end{aligned} \tag{122}$$

where the partial derivatives are computed at the steady state. Computing $S_1 = T$,

$$S_2 = \left| \begin{bmatrix} \frac{\partial f_3}{\partial P} & \frac{\partial f_3}{\partial s} \\ \frac{\partial f_4}{\partial P} & \frac{\partial f_4}{\partial s} \end{bmatrix} \right| + \left| \begin{bmatrix} \frac{\partial f_2}{\partial k} & \frac{\partial f_2}{\partial s} \\ \frac{\partial f_4}{\partial k} & \frac{\partial f_4}{\partial s} \end{bmatrix} \right| + \left| \begin{bmatrix} \frac{\partial f_2}{\partial k} & \frac{\partial f_2}{\partial P} \\ \frac{\partial f_3}{\partial k} & \frac{\partial f_3}{\partial P} \end{bmatrix} \right| \tag{123}$$
$$+ \left| \begin{bmatrix} \frac{\partial f_1}{\partial \mu} & \frac{\partial f_1}{\partial s} \\ \frac{\partial f_4}{\partial \mu} & \frac{\partial f_4}{\partial s} \end{bmatrix} \right| + \left| \begin{bmatrix} \frac{\partial f_1}{\partial \mu} & \frac{\partial f_1}{\partial P} \\ \frac{\partial f_3}{\partial \mu} & \frac{\partial f_3}{\partial P} \end{bmatrix} \right| + \left| \begin{bmatrix} \frac{\partial f_1}{\partial \mu} & \frac{\partial f_1}{\partial k} \\ \frac{\partial f_2}{\partial \mu} & \frac{\partial f_2}{\partial k} \end{bmatrix} \right|,$$

$$S_3 = \left| \begin{bmatrix} \frac{\partial f_2}{\partial k} & \frac{\partial f_2}{\partial P} & \frac{\partial f_2}{\partial s} \\ \frac{\partial f_3}{\partial k} & \frac{\partial f_3}{\partial P} & \frac{\partial f_3}{\partial s} \\ \frac{\partial f_4}{\partial k} & \frac{\partial f_4}{\partial P} & \frac{\partial f_4}{\partial s} \end{bmatrix} \right| + \left| \begin{bmatrix} \frac{\partial f_1}{\partial \mu} & \frac{\partial f_1}{\partial P} & \frac{\partial f_1}{\partial s} \\ \frac{\partial f_3}{\partial \mu} & \frac{\partial f_3}{\partial P} & \frac{\partial f_3}{\partial s} \\ \frac{\partial f_4}{\partial \mu} & \frac{\partial f_4}{\partial P} & \frac{\partial f_4}{\partial s} \end{bmatrix} \right| \tag{124}$$
$$+ \left| \begin{bmatrix} \frac{\partial f_1}{\partial \mu} & \frac{\partial f_1}{\partial k} & \frac{\partial f_1}{\partial s} \\ \frac{\partial f_2}{\partial \mu} & \frac{\partial f_2}{\partial k} & \frac{\partial f_2}{\partial s} \\ \frac{\partial f_4}{\partial \mu} & \frac{\partial f_4}{\partial k} & \frac{\partial f_4}{\partial s} \end{bmatrix} \right| + \left| \begin{bmatrix} \frac{\partial f_1}{\partial \mu} & \frac{\partial f_1}{\partial k} & \frac{\partial f_1}{\partial P} \\ \frac{\partial f_2}{\partial \mu} & \frac{\partial f_2}{\partial k} & \frac{\partial f_2}{\partial P} \\ \frac{\partial f_3}{\partial \mu} & \frac{\partial f_3}{\partial k} & \frac{\partial f_3}{\partial P} \end{bmatrix} \right|,$$

and $S_4 = D$, we obtain the sums of principal minors (109) to (112). ■

Proof of Proposition 6

The variables k, P and s are predetermined, while the multiplier μ isn't. We observe that $D = \lambda_1\lambda_2\lambda_3\lambda_4 < 0$. Let us show that at least one eigenvalue is real and positive.

There are three cases: (1) 4 real eigenvalues; (2) 2 real eigenvalues, say λ_1 and λ_2, and a pair of non-real and conjugate eigenvalues; (3) two pairs of non-real and conjugate eigenvalues, say (λ_1, λ_2) and (λ_3, λ_4).

(1) If $D = \lambda_1\lambda_2\lambda_3\lambda_4 < 0$, at least one eigenvalue is positive, otherwise $D \geq 0$.

(2) $\lambda_3\lambda_4 > 0$ and, therefore, $\lambda_1\lambda_2 < 0$, that is one real eigenvalue is positive.

(3) $\lambda_1\lambda_2 > 0$ and $\lambda_3\lambda_4 > 0$, that is $D > 0$, a contradiction.

The only possible cases are (1) and (2): at least one eigenvalue is real and positive, that is unstable.

Local indeterminacy of a four-dimensional system with three predetermined variables requires four stable eigenvalues. That is not the case in our model. Then, the equilibrium is locally determinate. ■

Proof of Proposition 7

Notice that, at the steady state, (k^*, P^*, s^*) does not depend on the shape of the utility function u, while u and, therefore, μ depend on (c, P), that is on (k^*, P^*, s^*).

In the case of the isoelastic utility function (22), (23) and (24) hold. Then, ε_{cP} is a constant, independent on (k, P, s), and (k^*, P^*, s^*) is independent on ε_{cP}.

Notice that ε_{cP}, that is η, only appears in the block D_2.

According to Proposition 16 in Bosi and Desmarchelier (2019), generically, a Hopf bifurcation arises if and only if

$$x \equiv \frac{S_3}{T} > 0, \tag{125}$$

and

$$S_2 = x + \frac{D}{x}. \tag{126}$$

Replacing (111) in (125), we get

$$D_2 = \frac{\Sigma_3 - Tx}{\alpha A_1 A_2 O_2}. \tag{127}$$

Replacing it in (110) and S_2 in (126), we find

$$(T - O_2)\, x^2 + (O_2 \Sigma_2 - \Sigma_3)\, x - DO_2 = 0. \tag{128}$$

Solving (128) for x, we obtain

$$x_- = \frac{-\left(O_2\Sigma_2 - \Sigma_3\right) - \sqrt{\left(O_2\Sigma_2 - \Sigma_3\right)^2 + 4DO_2\left(T - O_2\right)}}{2\left(T - O_2\right)}, \tag{129}$$

$$x_+ = \frac{-\left(O_2\Sigma_2 - \Sigma_3\right) + \sqrt{\left(O_2\Sigma_2 - \Sigma_3\right)^2 + 4DO_2\left(T - O_2\right)}}{2\left(T - O_2\right)}. \tag{130}$$

Noticing that $D < 0$ and $O_2 \leq 0$, two cases arise.

Case (1): $T - O_2 = \theta - a > 0$. Clearly, $x_- \leq 0 \leq x_+$ and hence, x_- does not satisfy inequality (125). Replacing x_+ in (127) and, finally, $T - O_2 = \theta - a$, we obtain (54).

Case (2): $T - O_2 = \theta - a < 0$. In this case, it is not possible to know the signs of x_- and x_+. However, in the particular case with no spontaneous change in mind: $\gamma_1(P) = \gamma_2(P) = 0$ (i.e. $C_1 = C_2 = 0$), we get $O_2 = 0$ and then,

$$\frac{S_3}{T} = \frac{\Sigma_3}{\theta - a}. \tag{131}$$

A Hopf bifurcation generically occurs when (126) holds if and only if $S_3/T > 0$, that is, since $\theta - a < 0$, if and only if $\Sigma_3 < 0$. Solving (126) for η gives (56). ■

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