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Clinton Gubong Gassi

CEPS
Centre for Economics at Paris-Saclay

école normale supérieure paris-saclay
université evry val-d'essonne université PARIS-SACLAY
université PARIS-SACLAY
Maison des Sciences de l'Homme PARIS-SACLAY

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Clinton Gubong Gassi*

Université Paris-Saclay, Université Evry Paris-Saclay , Centre for Economics at Paris-Saclay (CEPS), France

Abstract

This paper investigates the existence and characterization of majoritarian stable committees in multi-winner elections. We first establish a necessary and sufficient condition for stability and derive procedures for identifying stable committees and determining the minimum committee size for which stability exists. We then examine the relationship between stability and inclusion in polarized societies, where voters systematically favor candidates from their own group. Our results show that the possibility of achieving both stability and inclusive representation depends on the composition of society and the size of the elected committee. These findings highlight the tension between majoritarian stability and minority representation in collective decision-making.

Keywords: Multi-winner elections; Majority; Stability; Inclusion.
JEL classification: D71, D72.

1 Introduction

Multi-winner elections raise important challenges in social choice theory, where individuals must aggregate their preferences to select a fixed-size subset of alternatives from a larger set. This type of preference aggregation arises in various real-world situations, such as parliamentary elections, candidate selection for competitions, and the allocation of a fixed number of public goods to individuals or groups within society. This process is commonly referred to as committee selection. A substantial part of the literature on committee selection focuses on extending single-winner voting procedures to the multi-winner setting. More specifically, a central question is how voting procedures that ensure fairness and efficiency when selecting a single

*Corresponding author. Email: clinton.gassi@univ-evry.fr

alternative can preserve these properties when selecting a subset of alternatives. For an overview of this literature, we refer the reader to Elkind et al. (2017), Faliszewski et al. (2018, 2019), Kilgour (2010), Gehrlein (1985), Fishburn (1981), Kilgour and Marshall (2012), and Skowron et al. (2019). Among these contributions, Fishburn (1981) and Gehrlein (1985) are particularly relevant, as they propose different ways to extend the Condorcet principle, a well-known concept in social choice theory, to the selection of committees. Both approaches introduce the notion of a Condorcet committee as an extension of the Condorcet winner in single-winner elections. Under both approaches, a Condorcet committee is a fixed-size subset of alternatives that satisfies a form of majoritarian stability: no other subset of the same size can defeat it in a pairwise majority comparison. However, such a committee may or may not exist, depending on voters' preferences and, in particular, on the size of the committee. This raises the natural question of under what conditions a majoritarian stable committee exists.

In majoritarian democracies, the selection of a subset of candidates to represent the whole population may favor the interests of majority groups at the expense of minority groups. This concern is closely related to the classical notion of the "tyranny of the majority." John Stuart Mill (1860) argued that majority rule can lead to the oppression of minority factions, comparing the power exercised by a majority to that of a despot. This concern remains relevant in contemporary democratic societies, where political institutions increasingly face demands for greater diversity, equity, inclusion, and representation. For example, on June 25, 2021, U.S. President Joseph R. Biden issued an executive order aimed at strengthening the implementation of diversity, equity, inclusion, and accessibility in the federal workforce (The White House, 2021). Similarly, following the Conservative Party's victory in the 2019 U.K. general election, Prime Minister Boris Johnson formed a cabinet described as the most diverse in British political history, with the stated aim of addressing the challenges faced by the country's increasingly polarized communities (Bokhari, 2019). These examples illustrate a broader concern in collective decision-making: ensuring that the interests and characteristics of minority groups are not systematically overlooked when a set of candidates is selected to represent the whole population.

Motivated by these real-world examples, a natural question is whether democratic institutions provide sufficient incentives for decision makers to adopt inclusive selection procedures. Inclusive procedures may face limited majoritarian support, particularly when majority groups perceive the inclusion of minority groups as reducing their own representation. This tension may be particularly salient in socially fragmented societies, where individuals tend to identify strongly with distinct groups

or networks. At the same time, ensuring that different groups are represented in the selected set may reduce opposition to the collective decision and increase its acceptance among members of the society. This suggests a potential trade-off between inclusiveness and stability in fragmented societies. This observation raises the following question: under what conditions can a social planner design a voting procedure that is both inclusive and stable? Related questions have been studied in other collective decision-making settings, such as the allocation of scarce resources (see Nganmeni et al., 2022).

We address this question in the context of multi-winner elections, where a set of individuals have to choose a committee that optimally reflects their preferences.[1] In our framework, each voter's preferences are represented by a utility function defined over the subsets of available candidates. A voter prefers a subset S to a subset T if and only if the utility derived from S is greater than that derived from T. The contribution of this paper is twofold. First, we investigate the existence and characterization of majoritarian stable committees in multi-winner elections. We adopt a notion of stability according to which a committee is stable if no majority of voters strictly prefers another committee of the same size. Note that this definition is closely related to the definition of a Condorcet committee *à la Fishburn*(Fishburn, 1981). We establish necessary and sufficient conditions for a committee to be stable. This characterization yields a practical procedure for verifying stability and leads to an explicit determination of the set of stable committees for any given election. Furthermore, we identify the minimum committee size for which a stable committee exists, thereby clarifying how the size of the elected committee affects the possibility of reaching a stable collective outcome.

Second, we examine the relationship between majoritarian stability and inclusion in polarized societies. We consider a setting in which voters and candidates belong to two distinct groups, and individuals systematically favor candidates from their own group. This framework allows us to investigate whether a committee can simultaneously be stable under majority rule and guarantee representation for both groups. We establish conditions under which stability and inclusion can be achieved simultaneously, showing that the answer depends on the relative sizes of the groups and on the number of candidates to be selected. In particular, our results highlight how majoritarian considerations may prevent minority representation and how the size of the committee can affect the possibility of achieving a more inclusive outcome.

Our analysis contributes to the literature on multi-winner elections by connect-

[1]Note that a substantial body of research on multi-winner elections addresses the integration of diversity, inclusion, and representativeness into committee selection. We refer the reader to Aziz (2019), Bredereck et al. (2017), Diss et al. (2025), Ianovski (2022), and Relia (2021), among others.

ing the characterization of Condorcet-type stability with the problem of inclusive representation. This perspective is particularly relevant in political environments where the legitimacy of collective decisions depends not only on their resistance to majority opposition but also on the representation of different segments of society.

The remainder of the paper is organized as follows. Section 2 introduces the model and the main definitions. Section 3 establishes the main characterization results and develops procedures for identifying stable committees, including a result on the minimum committee size that guarantees stability. Section 4 examines the relationship between stability and inclusion in polarized societies and establishes conditions under which both properties can be satisfied. Section 5 concludes the paper.

2 Definitions and notations

For any set X, its cardinality is denoted by $|X|$, and its power set, that is, the set of all subsets of X, is denoted by $\mathcal{P}(X)$. For any $k \in \{1, \ldots, |X|\}$, let $\mathcal{P}_k(X)$ denote the set of all k-element subsets of X.

We consider a population $N = \{1, \ldots, n\}$ of n individuals (or voters), with $n \geq 2$, and a set $A = \{a_1, \ldots, a_m\}$ of m alternatives (or candidates), with $m \geq 3$. We adopt a utility-based framework in which each voter is endowed with a utility function defined over the power set $\mathcal{P}(A)$ of the set of candidates. Formally, each voter $i \in N$ has a utility function $u_i : \mathcal{P}(A) \to \mathbb{R}+$, which assigns to each subset $X \subseteq A$ a non-negative value $u_i(X)$ representing the utility that voter i derives from X. The utility function u_i induces an ordinal preference relation $\succsim_i$ on $\mathcal{P}(A)$, defined by $X \succsim_i Y$ if and only if $u_i(X) \geq u_i(Y)$, for all $X, Y \in \mathcal{P}(A)$. The relation $\succ_i$ denotes the strict part of $\succsim_i$, corresponding to strict inequality between utility values, while $\sim_i$ denotes its indifference part, corresponding to equality between utility values. The n-tuple $(\succsim_i)_{i \in N} = (\succsim_1, \succsim_2, \ldots, \succsim_n)$ is called the preference profile induced by the utility profile $(u_i)_{i \in N} = (u_1, u_2, \ldots, u_n)$.

Our objective is to identify a fixed-size subset of candidates, called a committee, that best reflects the voters' preferences. For any $k \in \{1, \ldots, m\}$, a committee of size k is a k-element subset of A, that is, an element of $\mathcal{P}_k(A)$. Given the population N, the set of candidates A, the committee size k, and the utility profile $(u_i)_{i \in N}$, we refer to the tuple $\mathcal{E} = \big(N, A, k, (u_i)_{i \in N}\big)$ as a multi-winner election.

We impose the following basic assumption on voters' utility functions: for all

$i \in N$ and all $X, Y, Z \in \mathcal{P}(A)$ such that $X \cap Y = X \cap Z = \emptyset$,

$$u_i(X \cup Y) \geq u_i(X \cup Z) \iff u_i(Y) \geq u_i(Z). \tag{1}$$

Assumption 1 can be interpreted as an independence from the common part condition: when comparing two sets that share a common subset X, a voter's comparison depends only on the respective additional subsets Y and Z, rather than on the common subset X. In other words, the presence of X does not affect the voter's relative evaluation of Y and Z. Without this assumption, the relative evaluation of Y and Z could depend on the presence of X, meaning that the same two subsets could be ranked differently depending on the context in which they are considered. In terms of ordinal preferences, Assumption 1 is equivalent to requiring that, for all $i \in N$ and all $X, Y, Z \in \mathcal{P}(A)$ such that $X \cap Y = X \cap Z = \emptyset$,

$$X \cup Y \succsim_i X \cup Z \iff Y \succsim_i Z.$$

A condition of this kind on voters' preferences over subsets of candidates can be found in Kamwa and Merlin (2018) and Kaymak and Sanver (2003).

A common example of utility functions is the class of additive utility functions. The utility functions $(u_i)_{i \in N}$ are said to be additive if, for each voter $i \in N$, there exists a weighting function $\delta_i \to \mathbb{R}_+$ that assigns a weight $\delta_i(a)$ to each candidate $a \in A$, such that, for every $X \in \mathcal{P}(A)$,

$$u_i(X) = \sum_{a \in X} \delta_i(a).$$

Note that, although the illustrative examples in this paper use additive preferences, the basic Assumption 1 on voters' utility functions is sufficient for our analysis.

In many democratic institutions, majority rule is a standard procedure for aggregating individual preferences when they diverge. The basic idea is to compare alternatives pairwise and identify those that receive the greatest support. In the single-winner setting, a candidate who cannot be defeated by another candidate in a pairwise majority comparison is known as a Condorcet winner. Several studies, including Gehrlein (1985), Fishburn (1981), and Kaymak and Sanver (2003), have investigated how this concept can be extended to the multi-winner setting, where the objects of comparison are committees rather than individual candidates. The notion of (Condorcet) stability introduced below is closer to the approach of Fishburn (1981), as it compares committees rather than individual candidates. Before the definition, let $\mathcal{M} = \{M \in \mathcal{P}(N) : |M| \geq |N \setminus M|\}$ be the set of majorities in

the population N.

Definition 1 *Let $\mathcal{E} = \big(N, A, k, (u_i)_{i\in N}\big)$ be a multi-winner election. A committee $W \in \mathcal{P}_k(A)$ is said to be (majoritarian) stable if there exists no committee $X \in \mathcal{P}_k(A)$ and no majority $M \in \mathcal{M}$ such that $u_i(X) > u_i(W)$ for every $i \in M$. Let $\mathcal{C}(N, A, k, (u_i)_{i\in N})$ denote the set of all majoritarian stable committees of size (k) with respect to the utility profile u.*

The following example illustrate that a stable committee might exist or not, depending on voters preferences and the committee size.

Example 1 *Consider a population $N = \{1, \ldots, 5\}$ and a set of candidates $A = \{a_1, \ldots, a_5\}$. Voters have additive preferences, with their utilities represented by the following matrix:*

	a_1	a_2	a_3	a_4	a_5
1	2	4	5	4	0
2	5	0	3	4	5
3	0	2	2	3	5
4	2	1	0	3	1
5	1	5	4	5	1

where the entry in row i and column a_j is the individual weight $\delta_i(a_j)$. recall that for every subset $X \subseteq A$, we have $u_i(X) = \sum_{a_j\in X} \delta_i(a_j)$.

First, consider committees of size $k = 2$. There are two stable committees:

$$\mathcal{C}(N, A, 2, (u_i)_{i\in N}) = \big\{\{a_2, a_4\}, \{a_4, a_5\}\big\}.$$

Indeed, neither committee can be defeated by another committee of size two supported by a majority of voters. For each of these committees, the largest coalition of voters strictly preferring another committee of the same size contains only two voters.

Now consider the committee size $k = 3$. In this case, it can be checked that the unique stable committee is $\{a_2, a_4, a_5\}$; that is, $\mathcal{C}(N, A, 3, (u_i)_{i\in N}) = \big\{\{a_2, a_4, a_5\}\big\}$.

However, the existence of a stable committee, is not guaranteed. To illustrate this, consider a second preference profile:

	a_1	a_2	a_3	a_4	a_5
1	0	2	1	2	1
2	3	2	3	2	0
3	1	1	0	0	3
4	0	2	0	2	1
5	3	0	1	1	1

and again let utilities be additive. We show that there is no stable committee of size $k = 2$.

Indeed, consider the following committees and their respective blocking committees:

W	X	*Voters strictly preferring* X *to* W
$\{a_1, a_2\}$	$\{a_2, a_5\}$	$\{1, 3, 4\}$
$\{a_1, a_3\}$	$\{a_1, a_2\}$	$\{1, 3, 4\}$
$\{a_1, a_4\}$	$\{a_2, a_5\}$	$\{1, 3, 4\}$
$\{a_1, a_5\}$	$\{a_1, a_2\}$	$\{1, 2, 4\}$
$\{a_2, a_3\}$	$\{a_4, a_5\}$	$\{3, 4, 5\}$
$\{a_2, a_4\}$	$\{a_1, a_2\}$	$\{2, 3, 5\}$
$\{a_2, a_5\}$	$\{a_2, a_4\}$	$\{1, 2, 4\}$
$\{a_3, a_4\}$	$\{a_1, a_3\}$	$\{2, 3, 5\}$
$\{a_3, a_5\}$	$\{a_1, a_2\}$	$\{2, 4, 5\}$
$\{a_4, a_5\}$	$\{a_1, a_5\}$	$\{2, 3, 5\}$

Every committee of size two is defeated by another committee of the same size supported by at least three voters. Consequently,

$$C(N, A, 2, (u_i)_{i \in N}) = \varnothing.$$

This example illustrates that the existence of a majoritarian stable committee depends on both the structure of voters' preferences and the size of the committee. In particular, stable committees may exist for some committee sizes but fail to exist for others.

3 Characterization and determination of stable committees

In this section, we establish some general results on the existence of stable committees by identifying their characteristic properties. Our first result follows directly from the definition of a stable committee. Recall that, by definition, a committee $W \in \mathcal{P}_k(A)$, with $1 \leq k \leq m$, is stable if, for every committee $W' \in \mathcal{P}_k(A)$ and every majority $M \in \mathcal{M}$, there exists at least one voter $i \in M$ such that $u_i(W') \leq u_i(W)$.

Before presenting the first result, we introduce the following notation. For any subset of candidates $T \in \mathcal{P}(A)$ and any majority $M \in \mathcal{M}$, let

$$\mathcal{B}(M, T) = \{S \in \mathcal{P}(A) : |S| = |T|,\ u_i(S) < u_i(T) \text{ for all } i \in M\}.$$

In other words, $\mathcal{B}(M,T)$ is the collection of all subsets of candidates having the same cardinality as T and such that every member of the majority M strictly prefers T to S.

Theorem 1 *Let $\mathcal{E} = \big(N, A, k, (u_i)_{i\in N}\big)$ be a multi-winner election, and let $W \in \mathcal{P}_k(A)$. The committee W is stable if and only if, for every $M \in \mathcal{M}$ and every $T \in \mathcal{P}(A)$, if there exists $S \subseteq W$ such that $S \in \mathcal{B}(M,T)$, then $W \cap T \neq \emptyset$.*

Proof. Assume that $W \in \mathcal{P}_k(A)$ is stable. Consider a majority $M \in \mathcal{M}$ and a subset of candidates $T \subseteq A$. Suppose that there exists $S \subseteq W$ such that $S \in \mathcal{B}(M,T)$. We show that $W \cap T \neq \emptyset$.

Suppose, by contradiction, that $W \cap T = \emptyset$. Then $W' = (W \setminus S) \cup T$ is a committee of size k, since $|S| = |T|$ and $T \cap W = \emptyset$. Moreover,

$$W \setminus W' = S \quad \text{and} \quad W' \setminus W = T.$$

Since $S \in \mathcal{B}(M,T)$, we have $u_i(T) > u_i(S)$ for every $i \in M$. By Assumption 1, it follows that

$$u_i\big(T \cup (W \cap W')\big) > u_i\big(S \cup (W \cap W')\big).$$

Consequently, $u_i(W') > u_i(W)$ for every $i \in M$, contradicting the stability of W. Hence, $W \cap T \neq \emptyset$.

Conversely, suppose that, for every $M \in \mathcal{M}$ and every $T \in \mathcal{P}(A)$, if there exists $S \subseteq W$ such that $S \in \mathcal{B}(M,T)$, then $W \cap T \neq \emptyset$. We show that W is stable.

Suppose, by contradiction, that W is not stable. Then there exist a committee $W' \in \mathcal{P}_k(A)$ and a majority $M \in \mathcal{M}$ such that $u_i(W') > u_i(W)$ for all $i \in M$. Let $S = W \setminus W'$ and $T = W' \setminus W$. Since $|W| = |W'|$, we have $|S| = |T|$. Furthermore,

$$W = S \cup (W \cap W') \quad \text{and} \quad W' = T \cup (W \cap W').$$

Thus, for every $i \in M$,

$$u_i\big(T \cup (W \cap W')\big) > u_i\big(S \cup (W \cap W')\big).$$

By Assumption 1, this implies that $u_i(T) > u_i(S)$ for every $i \in M$. Therefore, $S \in \mathcal{B}(M,T)$. Moreover, $S \subseteq W$ and $W \cap T = \emptyset$. This contradicts our initial assumption. Hence, W is stable. ∎

Theorem 1 provides a criterion for checking the stability of a committee. It states that a committee is stable if and only if, whenever a majority unanimously prefers some set of candidates T to another set S contained in the committee, the

committee must already contain at least one candidate from T. In other words, a stable committee cannot exclude all the candidates in a set that a majority unanimously prefers to an equally sized subset of the committee. The following example illustrates how this criterion can be applied.

Example 2 *Consider a population $N = \{1, 2, 3\}$ and a set of candidates $A = \{a_1, a_2, a_3\}$. Assume that the voters have additive utility functions that induce the following preferences over individual candidates:*

$$\begin{aligned} &\textit{Voter 1: } a_1 \succ_1 a_3 \succ_1 a_2; \\ &\textit{Voter 2: } a_2 \succ_2 a_3 \succ_2 a_1; \\ &\textit{Voter 3: } a_3 \succ_3 a_1 \sim_3 a_2. \end{aligned}$$

Suppose that the size of the committee to be selected is $k = 2$. Then,

$$\mathcal{P}_k(A) = \big\{\{a_1, a_2\}, \{a_1, a_3\}, \{a_2, a_3\}\big\}.$$

By Theorem 1, the committee $\{a_1, a_2\}$ is not stable. Indeed, take $M = \{1, 3\}$, $T = \{a_3\}$, and $S = \{a_2\}$. Both voters in M strictly prefer T to S, so $S \in \mathcal{B}(M, T)$. However, $\{a_1, a_2\} \cap T = \emptyset$, which violates the condition in Theorem 1.

Similarly, the committee $\{a_2, a_3\}$ is not stable. To see this, take $M = \{1, 3\}$, $T = \{a_1\}$, and $S = \{a_3\}$. Both voters in M strictly prefer T to S, so $S \in \mathcal{B}(M, T)$. However, $\{a_2, a_3\} \cap T = \emptyset$.

Now consider the committee $W = \{a_1, a_3\}$. The only nonempty subset of candidates that is disjoint from W is $T = \{a_2\}$. For $M \in \{\{1, 3\}, \{2, 3\}\}$, there is no $S \in \mathcal{B}(M, T)$ such that $S \subseteq W$. For $M = \{1, 2\}$, we can see that $\{a_3\} \in \mathcal{B}(M, \{a_1\})$ and $W \cap \{a_1\} \neq \emptyset$. Therefore, the unique stable committee is $W = \{a_1, a_3\}$; that is,

$$\mathcal{C}(N, A, k, (u_i)_{i \in N}) = \big\{\{a_1, a_3\}\big\}.$$

As illustrated in Example 2, the condition provided in Theorem 1 is particularly useful for showing that a committee W is not stable. Indeed, it suffices to identify two subsets of candidates, T and S, such that T is disjoint from W, S is contained in W, and a majority of voters strictly prefers T to S. However, verifying the stability of a committee using Theorem 1 can be more demanding, as it may require examining many subsets of candidates.

To simplify this verification, observe that, whenever T is disjoint from W, we have $T \subseteq A \setminus W$. Moreover, if $S \subseteq W$ and $|T| = |S|$, then $|T| \leq k$ and $|T| \leq m - k$.

Therefore, $|T| \leq \min\{k, m-k\}$. Let $\mu = \min\{k, m-k\}$ and define

$$\mathcal{K}(W) = \{T \subseteq A \setminus W : |T| \leq \mu\}.$$

Theorem 2 *Let $\mathcal{E} = \big(N, A, k, (u_i)_{i\in N}\big)$ be a multi-winner election and let $W \in \mathcal{P}_k(A)$. The committee W is stable if and only if, for every $S \subseteq W$ and every $T \in \mathcal{K}(W)$ such that $|T| = |S|$, we have $S \notin \mathcal{B}(M, T)$ for every $M \in \mathcal{M}$.*

Proof. Assume that W is stable. Consider $S \subseteq W$ and $T \in \mathcal{K}(W)$ such that $|T| = |S|$. We show that $S \notin \mathcal{B}(M, T)$ for every majority $M \in \mathcal{M}$. Suppose, by contradiction, that there exists $M \in \mathcal{M}$ such that $S \in \mathcal{B}(M, T)$. Since W is stable, Theorem 1 implies that $W \cap T \neq \emptyset$. However, $T \in \mathcal{K}(W)$ implies that $T \subseteq A \setminus W$, and hence $W \cap T = \emptyset$, a contradiction. Therefore, $S \notin \mathcal{B}(M, T)$ for every $M \in \mathcal{M}$.

Conversely, assume that, for every $S \subseteq W$ and every $T \in \mathcal{K}(W)$ such that $|T| = |S|$, we have $S \notin \mathcal{B}(M, T)$ for every $M \in \mathcal{M}$. We show that W is stable. Consider a majority $M \in \mathcal{M}$, a subset of candidates $T \subseteq A$, and assume that there exists $S \subseteq W$ such that $S \in \mathcal{B}(M, T)$. We have to show that $T \cap W \neq \emptyset$. By contradiction, assume that $T \cap W = \emptyset$. Then $T \subseteq A \setminus W$, which implies that $|T| \leq m - k$.

- If $T \in \mathcal{K}(W)$, then $|T| \leq \mu$. Since there exists $S \subseteq W$ such that $S \in \mathcal{B}(M, T)$, it follows that $|T| = |S|$ and and $u_i(T) > u_i(S)$ for all $i \in M$, which is a contradiction of the initial assumption.

- If $T \notin \mathcal{K}(W)$, have $|T| > \mu = \min\{k, m-k\}$. Since $|T| \leq m - k$, it follows that $k < |T| \leq m - k$. Thus, T contains more elements that W and no coalition $S \subseteq W$ can satisfy $|S| = |T|$. Therefore, there is no $M \in \mathcal{M}$ such that $S \in \mathcal{B}(M, T)$ and $S \subseteq W$, which contradicts the assumption. Hence, we conclude that W is a stable committee.

■

Theorem 2 provides a more convenient procedure for checking the stability of a committee. In particular, it suffices to compare subsets of W with subsets of $A \setminus W$ of the same cardinality, considering only subsets whose cardinality does not exceed μ. A committee is stable precisely when no such subset of W is strictly dominated by a corresponding subset of $A \setminus W$ in the eyes of any majority of voters.

While the previous theorems provide necessary and sufficient conditions for the stability of a committee, the following theorem characterizes the set of stable com-

mittees directly. To state this result, for every $S \in \mathcal{P}(A)$, let

$$\mathcal{D}(S) = \{T \in \mathcal{P}(A) : \exists M \in \mathcal{M},\ S \in \mathcal{B}(M, T)\},$$

and define

$$\mathcal{H} = \{W \in \mathcal{P}(A) : \forall S \subseteq W,\ \forall T \in \mathcal{D}(S),\ W \cap T \neq \emptyset\}.$$

In other words, $\mathcal{D}(S)$ is the collection of subsets of candidates that have the same cardinality as S and are strictly preferred to S by every voter in some majority. The set $\mathcal{H}$ consists of all subsets W of candidates such that no subset of candidates disjoint from W can be strictly preferred by a majority to a subset of W of the same cardinality.

Theorem 3 *Let $\mathcal{E} = \big(N, A, k, (u_i)_{i \in N}\big)$ be a multi-winner election. Then the set of stable committees of size k is given by*

$$\mathcal{C}(N, A, k, (u_i)_{i \in N}) = \mathcal{H} \cap \mathcal{P}_k(A).$$

Proof. Let $W \in \mathcal{C}(N, A, k, (u_i)_{i \in N})$ be a stable committee. By definition, $W \in \mathcal{P}_k(A)$. Consider any $S \subseteq W$ and any $T \in \mathcal{D}(S)$. By the definition of $\mathcal{D}(S)$, we have $|T| = |S| \leq k$. Moreover, there exists a majority $M \in \mathcal{M}$ such that $S \in \mathcal{B}(M, T)$. Since W is stable, Theorem 1 implies that $W \cap T \neq \emptyset$. Thus, $W \in \mathcal{H}$, and therefore

$$\mathcal{C}(N, A, k, (u_i)_{i \in N}) \subseteq \mathcal{H} \cap \mathcal{P}_k(A).$$

Conversely, consider $W \in \mathcal{H} \cap \mathcal{P}_k(A)$. We show that W is stable. By Theorem 1, it suffices to show that, for every $M \in \mathcal{M}$ and every $T \subseteq A$, if there exists $S \subseteq W$ such that $S \in \mathcal{B}(M, T)$, then $W \cap T \neq \emptyset$. Consider any such M, T, and S. Since $S \in \mathcal{B}(M, T)$, the definition of $\mathcal{D}(S)$ implies that $T \in \mathcal{D}(S)$. As $W \in \mathcal{H}$ and $S \subseteq W$, we obtain $W \cap T \neq \emptyset$. Hence, Theorem 1 implies that W is stable. Consequently,

$$\mathcal{H} \cap \mathcal{P}_k(A) \subseteq \mathcal{C}(N, A, k, (u_i)_{i \in N}),$$

which establishes the desired equality. ∎

It is worth noting that $\mathcal{H}$ characterizes stable committees of all possible sizes. In particular, the set of all candidates A belongs to $\mathcal{H}$, since every nonempty subset of A intersects A. Thus, $\mathcal{H}$ is nonempty. Thus, if stable committees are viewed as satisfactory outcomes for representing voters' preferences, a natural question is to

determine the minimum committee size that guarantees the existence of a stable committee.

Corollary 1 *Let* $k^* = \min\{|W| \in \mathcal{H},\ W \neq \emptyset\}$. *Then* $\mathcal{C}(N, A, k^*, (u_i)_{i\in N}) \neq \emptyset$, *and* $\mathcal{C}(N, A, k, (u_i)_{i\in N}) = \emptyset$ *for every* k *such that* $1 \leq k < k^*$.

Proof. By the definition of k^*, there exists a nonempty set $W \in \mathcal{H}$ such that $|W| = k^*$. Hence, $W \in \mathcal{H} \cap \mathcal{P}_{k^*}(A)$, and Theorem 3 implies that $\mathcal{C}(N, A, k^*, (u_i)_{i\in N}) \neq \emptyset$. Moreover, no set in $\mathcal{H}$ has cardinality strictly between 1 and $k^* - 1$. By Theorem 3, it follows that $\mathcal{C}(N, A, k^*, (u_i)_{i\in N}) = \emptyset$ for every k such that $1 \leq k < k^*$. ■

4 Inclusive and stable committees in polarized societies

In this section, we consider a polarized society in which individuals are divided into two disjoint groups according to their social, political, or economic ideologies. In particular, we focus on settings in which candidates are also members of the population, so that both the set of voters and the set of candidates can be partitioned into two groups, $N = N_1 \cup N_2$ and $A = A_1 \cup A_2$, respectively, with $N_1 \cap N_2 = \emptyset$ and $A_1 \cap A_2 = \emptyset$. We assume that individuals and candidates are located on a one-dimensional ideological axis and that, for each $j \in \{1, 2\}$, every candidate in A_j belongs to group N_j.

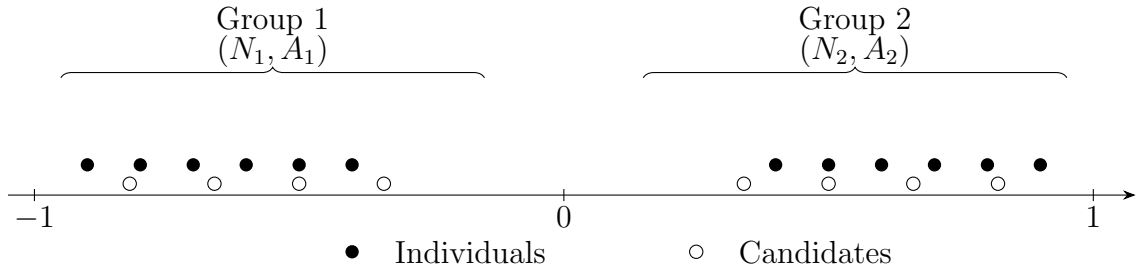


Figure 1: Individuals and candidates on the ideological axis.
Note. Individuals and candidates are represented on a one-dimensional ideological axis. Candidates in A_1 belong to group N_1, whereas candidates in A_2 belong to group N_2. The two groups occupy distinct regions of the ideological axis.

In this setting, each voter prefers any candidate from their own group to any candidate from the other group; that is, for every $a \in A_1$ and $b \in A_2$, we have $\{a\} \succ_i \{b\}$ for all $i \in N_1$ and $\{b\} \succ_i \{a\}$ for all $i \in N_2$.

Definition 2 *Let $\mathcal{E} = (N, A, k, (u_i)_{i\in N})$ be a multi-winner election. A committee $W \in \mathcal{P}(A)$ is said to be inclusive if $W \cap A_1 \neq \emptyset$ and $W \cap A_2 \neq \emptyset$.*

In other words, a committee is inclusive if each group in society is represented by at least one candidate. The following theorems provide necessary and sufficient conditions on the committee size for the existence of stable and inclusive committees, depending on the composition of the society.

Theorem 4 *Let $\mathcal{E} = (N, A, k, (u_i)_{i\in N})$ be a multi-winner election in a polarized society such that $N = N_1 \cup N_2$ and $A = A_1 \cup A_2$, with $N_1 \cap N_2 = \emptyset$ and $A_1 \cap A_2 = \emptyset$. If $N_1 \in \mathcal{M}$ and $N_2 \in \mathcal{M}$, then $\mathcal{C}(N, A, k, (u_i)_{i\in N}) \neq \emptyset$ if and only if $k = m$.*

Proof. We first show that no stable committee exists for any $k < m$. Let $W \in \mathcal{P}_k(A)$, where $k < m$. Since $W \neq A$, at least one of the inclusions $A_1 \subseteq W$ and $A_2 \subseteq W$ must fail. Moreover, since $k \geq 1$, the committee W contains at least one candidate from either A_1 or A_2. Without loss of generality, suppose that $A_1 \not\subseteq W$, and choose $a \in A_1 \setminus W$.

- If $W \cap A_2 \neq \emptyset$, choose $b \in W \cap A_2$ and consider the committee $W' = (W \setminus \{b\}) \cup \{a\}$. Since $a \notin W$ and $b \in W$, the two committees have the same cardinality. Moreover, for every $i \in N_1$, we have $u_i(\{a\}) > u_i(\{b\})$. By Assumption 1, it follows that
$$u_i\big((W \setminus \{b\}) \cup \{a\}\big) > u_i\big((W \setminus \{b\}) \cup \{b\}\big),$$
and hence $u_i(W') > u_i(W)$. Thus, every voter in N_1 strictly prefers W' to W. Since $N_1 \in \mathcal{M}$, this contradicts the stability of W.

- If $W \cap A_2 = \emptyset$, then $W \subseteq A_1$. Choose any $a \in W$ and any $b \in A_2$, and define $W^* = (W \setminus \{a\}) \cup \{b\}$. For every $i \in N_2$, we have $u_i(\{b\}) > u_i(\{a\})$. By Assumption 1, this implies that
$$u_i\big((W \setminus \{a\}) \cup \{b\}\big) > u_i\big((W \setminus \{a\}) \cup \{a\}\big),$$
so $u_i(W^*) > u_i(W)$. Thus, every voter in N_2 strictly prefers W^* to W. Since $N_2 \in \mathcal{M}$, this contradicts the stability of W.

We conclude that $\mathcal{C}(N, A, k, (u_i)_{i\in N}) = \emptyset$ for every $k < m$. Conversely, when $k = m$, the only committee is A, which is stable because no other committee of size m exists. Hence, $\mathcal{C}(N, A, m, (u_i)_{i\in N}) \neq \emptyset$, completing the proof. ∎

Theorem 4 shows that when both groups constitute a majority, no committee of size strictly smaller than m is stable. In this case, stability can be achieved only

by selecting all available candidates. Indeed, the committee A is stable because it contains every candidate from both groups and cannot be replaced by a different committee of the same size.

The following theorem examines the case in which only one group constitutes a majority.

Theorem 5 *Let $\mathcal{E} = (N, A, k, (u_i)_{i\in N})$ be a multi-winner election in a polarized society such that $N = N_1 \cup N_2$ and $A = A_1 \cup A_2$, with $N_1 \cap N_2 = A_1 \cap A_2 = \emptyset$. If $N_1 \in \mathcal{M}$ and $N_2 \notin \mathcal{M}$, then a stable and inclusive committee exists if and only if $k \geq |A_1| + 1$.*

Proof. Assume that $N_1 \in \mathcal{M}$ and $N_2 \notin \mathcal{M}$.

- If $k < |A_1| + 1$, that is, if $k \leq |A_1|$, let $W \in \mathcal{P}_k(A)$ be an inclusive committee. Clearly, $A_1 \not\subseteq W$, because $W \cap A_2 \neq \emptyset$ by inclusiveness and $|A_1| \geq k$. Let $a \in A_1 \setminus W$ and $b \in W \cap A_2$. Consider the committee $W' = (W \setminus \{b\}) \cup \{a\}$. For all $i \in N_1$, since $u_i(\{a\}) > u_i(\{b\})$, Assumption 1 implies that $u_i(W') > u_i(W)$. Therefore, $u_i(W') > u_i(W)$ for all $i \in N_1$. Since $N_1 \in \mathcal{M}$, W is not stable.

- If $k \geq |A_1| + 1$, we show that a stable and inclusive committee exists. Consider a committee $W \in \mathcal{P}_k(A)$ such that $A_1 \subsetneq W$ and W contains the $k - |A_1|$ candidates in A_2 closest to 0 on the ideological axis, as represented in Figure 1. Denote these candidates by $b_1, \ldots, b_{k-|A_1|}$. Since $A_1 \subsetneq W$ and $W \cap A_2 \neq \emptyset$, the committee W is inclusive.

  Moreover, for any subsets of candidates $S \subseteq W$ and $T \subseteq A \setminus W$ such that $|S| = |T|$, there is no voter $i \in N_1$ such that $u_i(T) > u_i(S)$, whereas $u_i(T) > u_i(S)$ for all $i \in N_2$. Indeed, for every voter $i \in N_1$, we have $b_j \succ_i b$ for every $b \in A_2 \setminus \{b_1, \ldots, b_{k-|A_1|}\}$. However, $N_2 \notin \mathcal{M}$. Hence, there is no majority $M \in \mathcal{M}$ such that $S \in B(M, T)$. By Theorem 2, we deduce that W is stable. Since $W \cap A_1 \neq \emptyset$ and $W \cap A_2 \neq \emptyset$, it follows that W is both stable and inclusive.

■

Theorem 5 establishes that, when N_1 constitutes a majority and N_2 does not, any stable and inclusive committee must contain more than $|A_1|$ candidates. The reason is that an inclusive committee of size at most $|A_1|$ necessarily excludes at least one candidate from A_1 while including a candidate from A_2. The voters in the majority group N_1 would then unanimously prefer replacing the latter candidate with the excluded candidate from A_1, contradicting stability.

5 Concluding remark

This paper has investigated the existence and characterization of majoritarian stable committees in multi-winner elections, with particular attention to the relationship between stability and inclusion in polarized societies.

We have established a necessary and sufficient condition for committee stability and developed procedures for identifying stable committees and determining the minimum committee size required for stability. We have also shown that the possibility of reconciling stability and inclusion depends on the composition of society and the size of the elected committee. In particular, our results highlight how majoritarian preferences can hinder minority representation and how committee size can affect the possibility of achieving inclusive outcomes.

These findings underscore the importance of considering stability and inclusion jointly when designing collective decision-making procedures. Future research could extend this framework to societies with more than two groups or explore alternative notions of Condorcet-wise stability, such as the one proposed by Gehrlein (1985).

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