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Abstract

This paper studies the allocation of a fixed number of an indivisible and homogeneous commodity among a set of agents. Each agent possesses a weak preference relation over the set of feasible assignments they can receive. Unlike classical models, we examine settings where agents hold asymmetric institutional power, allowing dissatisfied coalitions to challenge or reject a proposed allocation. In this framework, we introduce a concept of stability that generalizes Pareto optimality and ensures conflict avoidance. We establish necessary and sufficient conditions for the existence of stable allocations independently of agents' preferences, and characterize them when agent preferences are single-peaked or single-dipped.

Key words: Allocation problem, stability, blocking coalitions, single-peaked preferences, single-dipped preferences.

1 Introduction

We consider an allocation problem in which a fixed number of indivisible units of a homogeneous good must be distributed among a set of agents. Each agent has a weak preference relation over the quantities of the good that he or she may receive. Such problems arise in

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many situations in which a fixed amount of resources or obligations must be distributed without relying on price adjustments. Examples include the allocation of scarce goods, the assignment of tasks, the sharing of public-good costs, and the rationing of goods traded at fixed prices. As a simple illustration, consider a group of workers who have jointly agreed to complete a project requiring a fixed number of working hours. Each worker may have different preferences over the number of hours assigned to them, reflecting, for example, their trade-off between income and leisure or their individual time constraints.

This setting belongs to the broad literature on allocation problems. A major distinction in this literature concerns the divisibility of the resource. Some studies consider the allocation of an infinitely divisible commodity, as in [Barbera et al. \(1997\)](#), [Dietzenbacher and Tamura \(2024\)](#), and [Sprumont \(1991\)](#), whereas others study the allocation of indivisible commodities, as in [Arribillaga et al. \(2023\)](#), [Herrero and Martínez \(2011\)](#), and [Klaus \(2001\)](#). Another important distinction concerns the structure of individual preferences. A substantial part of the literature focuses on single-peaked preferences, including [Arribillaga et al. \(2023\)](#), [Barbera et al. \(1997\)](#), and [Sprumont \(1991\)](#), while other contributions study single-dipped preferences, including [Dietzenbacher and Tamura \(2023, 2024\)](#) and [Klaus et al. \(1997\)](#). These preference restrictions are particularly relevant for allocation problems because they allow the analysis of how agents' ideal or least-preferred quantities affect the properties of feasible allocations.

The present paper focuses on the allocation of an indivisible and homogeneous good. In contrast to models in which agents have preferences over different objects, such as [Fujinaka and Wakayama \(2018\)](#), [Ma \(1994\)](#), [Shapley and Scarf \(1974\)](#), and [Svensson \(1999\)](#), we consider a discrete division problem in which agents have preferences over the quantity of a common good that they receive. This type of allocation problem was introduced by [Herrero and Martínez \(2011\)](#) and has recently been studied by [Arribillaga et al. \(2023\)](#). In this literature, the main objective is generally normative: one seeks to identify allocations or allocation rules satisfying desirable properties such as Pareto optimality, strategy-proofness, fairness, or envy-freeness. Pareto optimality plays a central role in this analysis. An allocation is Pareto-optimal if there is no other feasible allocation that makes all agents weakly better off and at least one agent strictly better off. These studies provide important insights into the efficiency and fairness of resource allocation.

However, in many economic and political environments, agents do not have the same ability to influence collective decisions. Bargaining and political power may be unequally distributed, and some groups may possess sufficient influence to reject an allocation or to force a decision maker to reconsider it. Such asymmetries can turn allocation decisions into sources of economic, organizational, or political conflict (see [Collier and Hoeffler, 2005](#); [Frey, 1968](#); [Nganmeni et al., 2026](#); [Unfried et al., 2022](#)). An allocation may therefore be Pareto-optimal and yet remain vulnerable to a collective challenge by a sufficiently influential group. Conversely, a deviation that benefits some agents and harms others

may be feasible even when the agents who lose from the deviation are not sufficiently influential to prevent it.

This observation motivates our main question: how does the distribution of institutional influence affect the stability of resource allocations? To address this question, we introduce a collection of blocking coalitions, representing the groups of agents that possess sufficient institutional power to challenge an allocation. A coalition can challenge an allocation when another feasible allocation makes all its members weakly better off and at least one of them strictly better off. An allocation is then called stable if it cannot be challenged by any blocking coalition.

Our notion of stability generalizes the standard Pareto criterion to environments in which only some groups of agents have the ability to enforce a deviation. In particular, when the only blocking coalition is the grand coalition, stability coincides with Pareto optimality. More generally, stability takes into account both individual preferences and the institutional structure determining which coalitions can effectively challenge an allocation. We show that every stable allocation is Pareto-optimal (Proposition 4), but the converse need not hold. Thus, the relevant question is not only whether an allocation is efficient, but also whether it can withstand challenges from the coalitions that possess sufficient influence.

We first investigate the existence of stable allocations. Because the set of feasible allocations is finite, the absence of stable allocations is closely related to the possibility of forming cycles of successive challenges. We introduce the notion of a circuit in the influence structure and show that the absence of such circuits guarantees stability (Proposition 1). We then provide conditions linking the influence structure to the absence of circuits, allowing to characterize and explicitly identify environments where stability can be guaranteed independently of agents' preferences (Theorem 1 and Proposition 2). We also show that the existence and form of stable allocations can be highly sensitive to the preference profile when agents do not possess veto power (Proposition 3).

We next restrict attention to two important preference domains. For single-peaked preferences, we characterize stable allocations in terms of the agents' ideal quantities and the aggregate demands of blocking coalitions (Theorem 2). The characterization shows how resource scarcity interacts with institutional influence: when the aggregate ideal demand of a blocking coalition exceeds the available supply, stability requires the entire resource to remain within that coalition, whereas a coalition whose aggregate ideal demand can be fully satisfied must receive its members' ideal quantities. For single-dipped preferences, we obtain a different structure (Theorem 3). Stable allocations tend to concentrate the good among agents who benefit from receiving it or, when the good is undesirable, concentrate the associated burden on agents who lack effective blocking power. This result reveals a potential tension between collective stability and distributive fairness.

The analysis also illustrates the economic relevance of the framework. Under single-peaked preferences, we apply the characterization to the rationing of scarce health-care resources, where regions or institutions may differ both in their demand for treatment and in their ability to influence allocation decisions. Under single-dipped preferences, we consider the allocation of nuclear-waste management burdens, where all regions may prefer to avoid receiving the undesirable good but may have different institutional capacities to resist its allocation. These applications illustrate how stability depends jointly on resource scarcity, preferences, and the distribution of institutional influence.

The remainder of the paper is organized as follows. Section 2 introduces the model and the notion of blocking coalitions. Section 3 investigates the existence of stable allocations. Section 4 characterizes stable allocations under single-peaked and single-dipped preferences and presents two economic applications. Section 5 concludes and discusses directions for future research.

2 Model

A central authority (or planner) must allocate k indivisible units of a good among a set $N = \{1, 2, \dots, n\}$ of agents, where $n \geq 2$ and $k \geq 1$ is a positive integer. An *allocation* is a vector $x = (x_1, x_2, \dots, x_n) \in \{0, \dots, k\}^n$ such that $\sum_{i=1}^n x_i = k$. For each $i \in N$, we refer to x_i as agent i 's assignment under allocation x , and we denote by $\mathcal{A}$ the set of all allocations. Each agent $i \in N$ has a preference relation $\succsim_i$, which is a weak order over the set $\{0, \dots, k\}$ of possible assignments. A *preference profile* (or simply a *profile*) is a collection $R = (\succsim_i)_{i \in N}$ of the agents' individual preferences. For any two $t, t' \in \{0, \dots, k\}$, we write $t \succ_i t'$ if agent i strictly prefers t to t' , and $t \sim_i t'$ if agent i is indifferent between t and t' . Moreover, we assume that each agent's preference over assignments induces a preference over allocations, with no externalities: an agent's welfare from an allocation depends solely on the assignment he or she receives. Formally, for any two allocations $x, y \in \mathcal{A}$ and any agent $i \in N$,

$$x \succ_i y \Leftrightarrow x_i \succ_i y_i \quad \text{and} \quad x \sim_i y \Leftrightarrow x_i \sim_i y_i.$$

The organization's influence structure determines which nonempty groups of agents have blocking (or veto) power over potential allocations. We refer to such a group as a *blocking coalition*, and we denote by $\mathcal{B}$ the set of all blocking coalitions. We assume that $\mathcal{B}$ satisfies the following conditions:

- (1) $\mathcal{B} \neq \emptyset$ and $\emptyset \notin \mathcal{B}$;
- (2) for all $S \in \mathcal{B}$, if $S \subseteq T$, then $T \in \mathcal{B}$ (monotonicity).

Assumption (1) ensures the existence of at least one blocking coalition and rules out the

empty coalition as an influential group. Assumption (2) imposes a monotonicity condition: any coalition containing a blocking coalition is itself a blocking coalition. In particular, assumptions (1) and (2) imply that the grand coalition N is a blocking coalition. An agent $i \in N$ is said to have veto power if $\{i\} \in \mathcal{B}$. In our framework, the structure of blocking coalitions plays a central role in determining stability, as the existence of stable allocations depends directly on this structure. It is worth noting that we do not require the set of blocking coalitions to be *proper*. In other words, two blocking coalitions $S, T \in \mathcal{B}$ may be disjoint, i.e., $S \cap T = \emptyset$.

Throughout the paper, a *political economy* refers to the tuple $\mathcal{E} = (N, \mathcal{B}, k)$. Given a preference profile $R = (\succsim_i)_{i \in N}$, the tuple $(\mathcal{E}, R) \equiv (N, \mathcal{B}, k, R)$ is called an *allocation problem*.

Definition 1 *Given two allocations $x, y \in \mathcal{A}$ and a blocking coalition $C \in \mathcal{B}$, we say that x challenges y via C , denoted by $x \succ_C y$, if $x \succsim_i y$ for all $i \in C$ and $x \succ_i y$ for at least one $i \in C$.*

Given a preference profile $R = (\succsim_i)_{i \in N}$, an allocation $x \in \mathcal{A}$ is said to be R -stable if it cannot be challenged by any other allocation via any blocking coalition. Formally, x is R -stable if there is no $y \in \mathcal{A} \setminus \{x\}$ and no $C \in \mathcal{B}$ such that $y \succ_C x$. For any allocation problem $(\mathcal{E}, R)$, we denote by $\mathcal{C}(\mathcal{E}, R)$ the set of all R -stable allocations.

A political economy $\mathcal{E}$ is said to be stable if $\mathcal{C}(\mathcal{E}, R) \neq \emptyset$ for every preference profile R ; that is, for every preference profile R , there exists at least one R -stable allocation.

Thus, if $x \succ_C y$, the coalition C can be partitioned into two subgroups, S and T , such that all members of S strictly prefer x to y , while those in T are indifferent between x and y . The agents in S are those who actively support replacing y with x , whereas the agents in T do not oppose the proposal because it does not affect their welfare. Such a challenge can arise only if S is nonempty. Therefore, whenever we write a blocking coalition as $C = S \cup T$, we assume that $S \neq \emptyset$.

3 Existence of stable allocations

In this section, we address the following question: which configurations of the political economy guarantee stability? In other words, can the central planner be sure that, regardless of the voters' preferences, there always exists an allocation that cannot be challenged? We begin by introducing a key concept for answering this question.

Definition 2 *Let $\mathcal{E} = (N, \mathcal{B}, k)$ be a political economy. A circuit of length $p \in \mathbb{N}^*$ is a sequence $\widehat{S} = (S_t, T_t)_{t=1, \dots, p}$ such that, for every $t \in \{1, \dots, p\}$,*

$$S_t, T_t \subseteq N, \quad S_t \cup T_t \in \mathcal{B}, \quad S_t \neq \emptyset, \quad S_t \cap T_t = \emptyset,$$

and

$$\left(\bigcap_{l=1}^p (S_l \cup T_l) \right) \cap S_t = \emptyset.$$

We denote by $\Gamma(\mathcal{E})$ the set of all circuits of $\mathcal{E}$. If $\Gamma(\mathcal{E}) \neq \emptyset$, we define $\nu(\mathcal{E}) = \min_{\widehat{\mathcal{S}} \in \Gamma(\mathcal{E})} |\widehat{\mathcal{S}}|$, where $|\widehat{\mathcal{S}}|$ denotes the length of the circuit $\widehat{\mathcal{S}}$.

A circuit can be interpreted as a closed loop of political or institutional power. It captures a cycle in which one blocking coalition can challenge an allocation, another blocking coalition can challenge the resulting alternative, and so on, eventually returning to the initial allocation. Such a cycle prevents any allocation from being stable.¹

Proposition 1 *Let $\mathcal{E} = (N, \mathcal{B}, k)$ be a political economy. If $\Gamma(\mathcal{E}) = \emptyset$, then $\mathcal{E}$ is stable.*

Proof. We proceed by contradiction. Assume that $\Gamma(\mathcal{E}) = \emptyset$ and $\mathcal{E}$ is not stable; that is, there exists a preference profile $R = (\succsim_i)_{i \in N}$ such that $\mathcal{C}(\mathcal{E}, R) = \emptyset$.

Let $\mathcal{A}$ be the set of all feasible allocations. Since both n and k are finite, $|\mathcal{A}| < \infty$ ². By assumption, every allocation in $\mathcal{A}$ is challenged. Therefore, for every $x^1 \in \mathcal{A}$, there exists another allocation $x^2 \in \mathcal{A}$ and a pair (S_1, T_1) of subsets of agents, such that $S_1 \cup T_1 \in \mathcal{B}$ and:

$$x^2 \succsim_i x^1 \forall i \in S_1, \quad \text{and} \quad x^2 \sim_j x^1 \forall j \in T_1.$$

Since x^2 is also challenged, we can find another allocation x^3 , and a pair (S_2, T_2) , such that:

$$x^3 \succsim_i x^2 \forall i \in S_2, \quad \text{and} \quad x^3 \sim_j x^2 \forall j \in T_2.$$

Repeating this reasoning, we construct a finite sequence of allocations $(x^1, x^2, \dots, x^m) \subseteq \mathcal{A}$ such that $x^{t+1} \succsim_{S_t \cup T_t} x^t$ for all $t \in \{1, \dots, m\}$. Since $\mathcal{A}$ is finite, there exist indices $1 \leq r < s \leq m$ such that $x^r = x^s$. Thus, we obtain a cycle of allocations challenge:

$$x^r \rightarrow x^{r+1} \rightarrow \dots \rightarrow x^{s-1} \rightarrow x^r$$

This cycle allows to derive the sequence $\{(S_t, T_t)\}_{t=r}^{s-1}$, which forms a circuit. Indeed, for each $t = 1, \dots, s-1$, we have:

- Each $S_t \cup T_t \in \mathcal{B}$,
- $S_t \neq \emptyset$,

¹The parameter $\nu(\mathcal{E})$ serves as a natural analogue of the well-known Nakamura number (Nakamura, 1979). While the classical Nakamura number characterizes the threshold at which cycles can arise in voting systems, $\nu(\mathcal{E})$ captures the minimum length of a sequence of blocking coalitions that can generate a preference cycle in our political economy.

²Note that the cardinality of $\mathcal{A}$ is given by $|\mathcal{A}| = \frac{(n+k-1)!}{(n-1)! \times k!}$

- $S_t \cap T_t = \emptyset$,
- Because of the strict preferences of agents in S_t , the cycle returns to the initial allocation, and implies that $S_t \cap \left(\bigcap_{l=r}^{s-1} (S_l \cup T_l)\right) = \emptyset$. Otherwise, there would exist an agent i belonging to this latter set and for this agent it would hold that $x^r \succ_i x^r$ which is a contradiction.

The above conditions meet the definition of a circuit. Thus, we have constructed an element of $\Gamma(\mathcal{E})$, contradicting our assumption that $\Gamma(\mathcal{E}) = \emptyset$. Hence, $\mathcal{E}$ is stable. $\blacksquare$

According to Proposition 1, a political economy is stable whenever its influence structure contains no circuit. In other words, instability is directly related to the possibility of forming cycles of successive challenges. The following theorem provides a complete characterization of stability in terms of the configuration of blocking coalitions.

Theorem 1 *Let $\mathcal{E} = (N, \mathcal{B}, k)$ be a political economy. The following assertions are equivalent:*

- (i) $\mathcal{C}(\mathcal{E}, R) \neq \emptyset$ for all preference profile R ;
- (ii) the set $\mathcal{B}$ is totally ordered by inclusion: for any $C, D \in \mathcal{B}$, either $C \subseteq D$ or $D \subseteq C$;
- (iii) $\Gamma(\mathcal{E}) = \emptyset$.

Proof. We write $C_t = S_t \cup T_t$ for each pair in a proposed circuit. By Definition 1, $S_t \neq \emptyset$, $S_t \cap T_t = \emptyset$, $C_t \in \mathcal{B}$, and

$$S_t \cap \bigcap_{\ell=1}^p C_\ell = \emptyset \quad \text{for every } t = 1, \dots, p. \quad (1)$$

There does not exist a circuit of length one, since $S_1 \cap (S_1 \cup T_1) = S_1 \neq \emptyset$. Whenever a circuit exists, its minimal length is two i.e. $\nu(\mathcal{E}) = 2$.

(iii) $\Rightarrow$ (ii). By contraposition, assume that there exist incomparable elements $C, D \in \mathcal{B}$, that is, neither $C \subseteq D$ nor $D \subseteq C$. Both $C \setminus D$ and $D \setminus C$ are therefore nonempty. Define

$$S_1 = C \setminus D, \quad T_1 = C \cap D, \quad S_2 = D \setminus C, \quad T_2 = C \cap D.$$

These sets satisfy $S_1 \cap T_1 = S_2 \cap T_2 = \emptyset$ and $S_1 \cup T_1 = C$, $S_2 \cup T_2 = D$, so the coalition and nonemptiness requirements hold. Furthermore,

$$(C \cap D) \cap S_1 = (C \cap D) \cap (C \setminus D) = \emptyset, \quad (C \cap D) \cap S_2 = (C \cap D) \cap (D \setminus C) = \emptyset.$$

Since $C \cap D = \bigcap_{\ell=1}^2 C_\ell$, condition (1) holds for both indices. Thus $((S_1, T_1), (S_2, T_2))$ is a circuit of length two. This contradicts $\Gamma(\mathcal{E}) = \emptyset$, and proves (iii) $\Rightarrow$ (ii).

(ii) $\Rightarrow$ (iii). Assume that any two members of $\mathcal{B}$ are comparable by inclusion. Consider any finite sequence of candidate pairs $((S_t, T_t))_{t=1}^p$ satisfying all requirements of Definition 1 except possibly (1). Because the coalitions $C_1, \dots, C_p$ are nested, one of them, say C_{t_0} , is contained in all the others. Hence $\bigcap_{\ell=1}^p C_\ell = C_{t_0}$. Yet $\emptyset \neq S_{t_0} \subseteq C_{t_0}$, and consequently

$$S_{t_0} \cap \bigcap_{\ell=1}^p C_\ell = S_{t_0} \cap C_{t_0} = S_{t_0} \neq \emptyset.$$

This violates (1); therefore $\Gamma(\mathcal{E}) = \emptyset$. The two implications prove (ii) $\Leftrightarrow$ (iii).

(ii) $\Rightarrow$ (i). Assume (ii) and fix any admissible profile R . Since N is finite, so are $\mathcal{B}$ and $\mathcal{A}$. List the distinct blocking coalitions in increasing order:

$$\mathcal{B} = \{C_1, \dots, C_m\}, \quad C_1 \subsetneq C_2 \subsetneq \dots \subsetneq C_m = N.$$

For each i , the weak order $\succsim_i$ partitions $\{0, \dots, k\}$ into finitely many indifference classes, ordered from worst to best. Assign to every quantity q the ordinal number $u_i(q)$ of its class. Thus, for all q, r ,

$$q \succsim_i r \Leftrightarrow u_i(q) \geq u_i(r), \quad q \succ_i r \Leftrightarrow u_i(q) > u_i(r).$$

Such u always exist, for example $u_i(p) = -|\{x : x \succ_i q\}|$.

For each $j = 1, \dots, m$ and $x \in \mathcal{A}$, define $U_j(x) = \sum_{i \in C_j} u_i(x_i)$. The finite set of vectors $\{(U_1(x), \dots, U_m(x)) : x \in \mathcal{A}\}$ has a lexicographic maximum. Choose an allocation x^* attaining it.

Suppose, for a contradiction, that some $y \in \mathcal{A}$ challenges x^* via a coalition C_j . By the definition of a challenge,

$$u_i(y_i) \geq u_i(x_i^*) \quad (i \in C_j), \quad u_{i_0}(y_{i_0}) > u_{i_0}(x_{i_0}^*) \quad \text{for some } i_0 \in C_j.$$

Summing over C_j gives $U_j(y) > U_j(x^*)$. For each $h < j$, the inclusion $C_h \subsetneq C_j$ and the individual weak inequalities imply $U_h(y) \geq U_h(x^*)$. Consider the first index $r \leq j$ at which the two vectors differ; it exists because their j th coordinates differ. For every $h < r$ the coordinates coincide, while $U_r(y) > U_r(x^*)$ (the inequality is strict at the first difference). Therefore the vector corresponding to y is lexicographically larger than that of x^* , contradicting the choice of x^* . No challenge exists, so x^* is stable. Since R was arbitrary, (ii) implies (i).

(i) $\Rightarrow$ (ii). We proceed by contraposition, assuming that there exist incomparable elements $C, D \in \mathcal{B}$. Both set $C \setminus D$ and $D \setminus C$ are non empty. Consider agents $a \in C \setminus D$ and $b \in D \setminus C$, who are distinct because $(C \setminus D) \cap (D \setminus C) = \emptyset$. Specify their preferences by $u_a(q) = u_b(q) = q$ for every $q \in \{0, \dots, k\}$. For each $i \notin \{a, b\}$, set $u_i(q) = 0$ for all q . These functions define complete and transitive individual preferences on quantities, as

required by the model. In particular, a and b strictly prefer larger assignments, while all other agents are indifferent.

Let $x \in \mathcal{A}$ be arbitrary. If $x_a < k$, let y be the feasible allocation with $y_a = k$ and $y_i = 0$ for every $i \neq a$. Since $a \in C$ and $u_a(y_a) = k > x_a = u_a(x_a)$, agent a strictly benefits. Every agent in $C \setminus \{a\}$ is indifferent between all quantities: the only other agent whose preferences depend on the quantity received is b , and $b \notin C$. Thus $u_i(y_i) = u_i(x_i)$ for all $i \in C \setminus \{a\}$, and y challenges x via C .

If $x_a = k$, feasibility and $a \neq b$ imply $x_b = 0$. Let z be the feasible allocation with $z_b = k$ and $z_i = 0$ for all $i \neq b$. Since $b \in D$, $u_b(z_b) = k > 0 = u_b(x_b)$. Every agent in $D \setminus \{b\}$ is indifferent between all quantities: the only other agent whose preferences depend on the quantity received is a , and $a \notin D$. Hence every member of $D \setminus \{b\}$ is indifferent, and z challenges x via D . Both cases cover every $x \in \mathcal{A}$, so this profile has no stable allocation. This contradicts (i) and proves (i) $\Rightarrow$ (ii). ■

Remark 1 *It is worth noting that we can also establish the following result, which is analogous to the result in Nakamura (1979): $\mathcal{C}(\mathcal{E}, R) \neq \emptyset$ for every preference profile R if and only if $|\mathcal{A}| < \nu(\mathcal{E})$. However, it follows from the proof of Theorem 1 that $\Gamma(\mathcal{E}) \neq \emptyset$ implies that the minimum circuit length is $\nu(\mathcal{E}) = 2$. Since $n \geq 2$ and $k \geq 1$, we necessarily have $|\mathcal{A}| \geq 2$, and therefore the condition $|\mathcal{A}| < \nu(\mathcal{E})$ can never be satisfied.*

While Theorem 1 provides a general necessary and sufficient condition for stability, the following proposition gives an explicit identification of the political economies in which stability is guaranteed.

Proposition 2 *Let $\mathcal{E} = (N, \mathcal{B}, k)$ be a political economy. The following statements are equivalent:*

1. $\mathcal{C}(\mathcal{E}, R) \neq \emptyset$ for every preference profile R ;
2. $\mathcal{B} = \{N\}$, or there exists $i \in N$ such that $\mathcal{B} = \{N \setminus \{i\}, N\}$.

Proof. By Theorem 1, statement (1) holds if and only if $\mathcal{B}$ is totally ordered by inclusion. Suppose that $\mathcal{B}$ is totally ordered, and let M be the smallest blocking coalition. If $|N \setminus M| \geq 2$, choose distinct agents $a, b \in N \setminus M$. By the monotonicity of $\mathcal{B}$, both $M \cup \{a\}$ and $M \cup \{b\}$ belong to $\mathcal{B}$. However, these two coalitions are incomparable by inclusion, which contradicts the assumption that $\mathcal{B}$ is totally ordered. Hence, $|N \setminus M| \leq 1$. If $M = N$, then $\mathcal{B} = N$. Otherwise, $M = N \setminus i$ for some $i \in N$, and monotonicity implies that $\mathcal{B} = \{N \setminus \{i\}, N\}$.

Conversely, each of these two families is totally ordered by inclusion. Therefore, Theorem 1 implies statement (1). ■

We now turn to an allocation problem $(\mathcal{E}, R)$, that is, a political economy in which agents have expressed their preferences. As illustrated by the following example, the stability of an allocation can be highly sensitive to agents' preferences.

Example 1 Consider an allocation problem $(N, \mathcal{B}, k, R)$ with $N = \{1, 2\}$, $k = 2$, $\mathcal{B} = \{N\}$, and the preference profile $R = (\succsim_i)_{i=1,2}$ defined by:

$$\text{Agent 1: } 1 \succ_1 2 \succ_1 0$$

$$\text{Agent 2: } 1 \succ_2 2 \succ_2 0$$

It can be checked that the allocation $x = (1, 1)$ is stable. Indeed, $y = (2, 0)$ cannot challenge x , since both agents prefer x to y ($1 \succ_1 2$ for agent 1 and $0 \succ_2 1$ is false for agent 2). The same holds for the allocation $z = (0, 2)$: Agent 1 prefers x to z ($1 \succ_1 0$), so z does not challenge x .

Now consider the preference profile $R' = (\succsim'_i)_{i=1,2}$ defined by

$$\text{Agent 1: } 2 \succ'_1 1 \succ'_1 0$$

$$\text{Agent 2: } 0 \succ'_2 1 \succ'_2 2$$

The allocation x is not R' -stable, since it is challenged by $y = (2, 0)$ via the grand coalition N .

Nevertheless, as stated in the following proposition, the presence of agents with veto power provides a trail to identify a stable allocation.

Proposition 3 Let $(N, \mathcal{B}, R, k)$ be an allocation problem and let x be an allocation. If x is stable, then any agent with a veto power, if it exists, receives his/her most preferred assignment in x . That is, if i is a veto power agent, the $x_i \succsim_i q$, for all $q \in \{0, \dots, k\}$.

Proof. Let $i \in N$ be a veto agent, i.e., $\{i\} \in \mathcal{B}$. Suppose x_i is not the i 's most preferred quantity. Then there exists some quantity $q \in \{0, 1, \dots, k\}$ such that $q \succ_i x_i$. We construct a new allocation y as follows:

- Set $y_i = q$.
- If $q > x_i$, we need to transfer $(q - x_i)$ goods to i . Since $\sum_{j \neq i} x_j = k - x_i \geq q - x_i$ (because $q \leq k$), we can reduce the assignments of other agents to obtain the required goods.
- If $q < x_i$, we have $(x_i - q)$ excess goods from i to distribute to other agents. Adjust the assignments of other agents to maintain feasibility: $\sum_{i=1}^n y_i = k$.

The resulting allocation y satisfies $y_i = q \succ_i x_i$. Since $\{i\} \in \mathcal{B}$ is a blocking coalition and i strictly prefers y to x , x is challenged by y via coalition $\{i\}$. Therefore, x is not stable. ■

Remark 2 *Example 1 and Proposition 3 highlight a relevant feature of the model: without the protective barrier of veto power, stability becomes highly sensitive to the preference structure. Even with the same agents, resources, and the same influence structure, different preference profiles can lead to opposite stability outcomes for the same allocation.*

Proposition 3 states that when there exists agents with a veto power, a stable allocation must be sought among those yielding the best assignment to veto agents. However, Example 2 below shows that the reverse of the statement of Proposition 3 is not true; that is, an allocation can yield then the top assignment to veto agents without being stable.

Example 2 *Consider the allocation problem $(\mathcal{E}, R) \equiv (N, \mathcal{B}, k, R)$ with $N = \{1, 2, 3\}$, $k = 2$, $\mathcal{B} = \{\{1\}, \{1, 2\}, \{1, 3\}, \{1, 2, 3\}\}$, and preference profile R given by:*

Agent 1 (veto): $1 \sim_1 2 \succ_1 0$;

Agent 2: $2 \succ_2 1 \succ_2 0$;

Agent 3: $1 \succ_3 0 \succ_3 2$.

Consider the allocation $x = (1, 1, 0)$. Agent 1 receives a most-preferred assignment, yet it can be checked that x is challenged by the allocation $y = (1, 0, 1)$ via the blocking coalition $C = \{1, 3\}$.

Obviously, the equivalence holds when every agent has veto power.

Corollary 1 *If all individuals have veto power, i.e., $\{i\} \in \mathcal{B}$ for all $i \in N$, then an allocation x is stable if and only if it gives each agent a most-preferred assignment; that is, x is stable $\Leftrightarrow x_i \succsim_i q$ for all $i \in N$ and all $q \in \{0, \dots, k\}$.*

Proof. By Proposition 3, if x is stable, then $x_i \succsim_i q$ for all $i \in N$ and all $q \in \{0, \dots, k\}$. Conversely, suppose that $x_i \succsim_i q$ for all $i \in N$ and all $q \in \{0, \dots, k\}$. Then no agent strictly prefers any feasible assignment to the assignment he or she receives in x . Hence, no allocation can challenge x via a singleton coalition i . Since every agent has veto power, this implies that x is stable. ■

In the classical framework without any influence structure within the economy, the most studied property of an allocation is Pareto optimality (see, for instance, Dietzenbacher and Tamura, 2024, Klaus et al., 1997, and Klaus, 2001). An allocation x is Pareto-optimal if there is no other allocation y such that $y_i \succsim_i x_i$ for all $i \in N$ and $y_i \succ_i x_i$ for at

least one agent $i \in N$. This property is particularly relevant when the political influence of agents does not matter and the only blocking coalition is the grand coalition, that is, the set of all agents. However, in the setting studied in this paper, Pareto optimality is no longer sufficient to capture the relevant notion of stability, since we are concerned only with the groups of agents that can challenge an allocation. The stability concept introduced in this paper generalizes Pareto optimality, as the two concepts coincide when the set of blocking coalitions is reduced to $\{N\}$. The following proposition formally establishes this generalization.

Proposition 4 *Let $(N, \mathcal{B}, R, k)$ be an allocation problem and let $x \in \mathcal{A}$ be an allocation. If x is R -stable, then x is Pareto-optimal.*

Proof. The proof is straightforward. Since x is R -stable, there is no $y \in \mathcal{A}$ and $C \in \mathcal{B}$ such that $y \succ_C x$. In particular, there cannot exist an allocation $y \in \mathcal{A}$ such that $y \succ_N x$, since $N \in \mathcal{B}$. Therefore, x is Pareto-optimal. ■

4 Characterization of stable allocations on restricted domains

As mentioned in the introductory section, most papers in the literature on allocation problems focus on the domains of single-peaked and single-dipped preferences. This is largely due to their contextual relevance and economic applications. This section is devoted to characterizing stable allocations on these preference domains.

4.1 Stability with single-peaked preferences

Single-peaked preferences were first introduced by Black (1948). Roughly speaking, single-peaked preferences arise when each agent has an ideal quantity, referred to as their “peak,” and their welfare decreases as their assignment moves farther away from this peak. Formally, a preference profile $R = (\succsim_i)_{i \in N}$ is a profile of single-peaked preferences if, for each agent $i \in N$, there exists $p_i \in 0, \dots, k$ such that, for all $q, r \in 0, \dots, k$,

$$r < q \leq p_i \text{ or } p_i \leq q < r \Rightarrow q \succsim_i r.$$

A profile of single-peaked preferences R can then be identified by the vector of peaks $(p_1, \dots, p_n)$. The following theorem provides a complete characterization of stable allocations under single-peaked preferences.

Theorem 2 *Let $(N, \mathcal{B}, R, k)$ be an allocation problem where all agents have strictly single-peaked preferences on $\{0, \dots, k\}$, with peaks $p_1, \dots, p_n$, and let $x \in \mathcal{A}$. Then x is stable if and only if, for every blocking coalition $C \in \mathcal{B}$, the applicable condition holds:*

(i) If $\sum_{i \in C} p_i > k$, then $x_i \leq p_i$ for every $i \in C$, and $x_j = 0$ for every $j \notin C$.

(ii) If $\sum_{i \in C} p_i \leq k$ and either $C \neq N$ or $\sum_{i \in C} p_i = k$, then $x_i = p_i$ for every $i \in C$, and consequently

$$\sum_{j \notin C} x_j = k - \sum_{i \in C} p_i.$$

(iii) If $C = N$ and $\sum_{i \in N} p_i < k$, then $x_i \geq p_i$ for every $i \in N$.

Proof. ($\Rightarrow$) Assume that x is stable, and let $C \in \mathcal{B}$ be any blocking coalition. We consider three cases.

Case 1: $\sum_{i \in C} p_i > k$. Suppose, by contradiction, that there exists $i_0 \in C$ with $x_{i_0} > p_{i_0}$. Since

$$\sum_{i \in C} p_i > k \geq \sum_{i \in C} x_i,$$

there exists $h \in C$ with $x_h < p_h$. Transfer one unit from i_0 to h , leaving every other assignment unchanged. The resulting allocation is feasible. By single-peakedness, both i_0 and h strictly improve, while every other member of C is indifferent. This contradicts stability. Thus $x_i \leq p_i$ for every $i \in C$.

Now suppose that some $j_0 \notin C$ receives $x_{j_0} > 0$. Since $\sum_{i \in C} p_i > k$ and $\sum_{i \in C} x_i \leq k$, there exists $h \in C$ with $x_h < p_h$. Transfer one unit from j_0 to h . This feasible allocation strictly improves h and leaves every other member of C indifferent. It therefore challenges x via C , a contradiction. Hence $x_j = 0$ for every $j \notin C$.

Case 2: $\sum_{i \in C} p_i \leq k$ and either $C \neq N$ or $\sum_{i \in C} p_i = k$. There exists a feasible allocation y such that $y_i = p_i$ for every $i \in C$. Indeed, if $C \neq N$, the remaining $k - \sum_{i \in C} p_i$ units can be assigned to an agent outside C ; if $C = N$, this remaining quantity is zero. If $x_{i_0} \neq p_{i_0}$ for some $i_0 \in C$, then y makes every member of C weakly better off and i_0 strictly better off. This contradicts stability. Thus $x_i = p_i$ for every $i \in C$, and feasibility gives

$$\sum_{j \notin C} x_j = k - \sum_{i \in C} p_i.$$

Case 3: $C = N$ and $\sum_{i \in N} p_i < k$. Suppose that $x_{i_0} < p_{i_0}$ for some $i_0 \in N$. Since

$$\sum_{i \in N} x_i = k > \sum_{i \in N} p_i,$$

there exists $h \in N$ with $x_h > p_h$. Transfer one unit from h to i_0 . Both agents strictly improve, and all others are indifferent. As $N \in \mathcal{B}$, this contradicts stability. Therefore $x_i \geq p_i$ for every $i \in N$.

($\Leftarrow$) Assume that the applicable conditions hold for every $C \in \mathcal{B}$. Suppose, by contradiction, that some allocation y challenges x via a blocking coalition C . Thus $y_i \succeq_i x_i$

for all $i \in C$, with a strict preference for at least one member of C . Again, consider the three cases.

Case 1: $\sum_{i \in C} p_i > k$. By condition (i), $x_i \leq p_i$ for every $i \in C$, and the coalition receives all k units:

$$\sum_{i \in C} x_i = k.$$

Because preferences strictly increase up to the peak, $y_i \succeq_i x_i$ implies $y_i \geq x_i$ for every $i \in C$. A strict improvement for at least one member implies a strict inequality for at least one such assignment. Consequently,

$$\sum_{i \in C} y_i > \sum_{i \in C} x_i = k,$$

which is not possible.

Case 2: $\sum_{i \in C} p_i \leq k$ and either $C \neq N$ or $\sum_{i \in C} p_i = k$. By condition (ii), every member of C receives their unique peak: $x_i = p_i$ for all $i \in C$. No member can strictly prefer their assignment under y . Hence y cannot challenge x via C .

Case 3: $C = N$ and $\sum_{i \in N} p_i < k$. By condition (iii), $x_i \geq p_i$ for every $i \in N$. Because preferences strictly decrease above the peak, $y_i \succeq_i x_i$ implies $y_i \leq x_i$ for every $i \in N$. If some agent strictly improves, at least one of these inequalities is strict. It follows that

$$\sum_{i \in N} y_i < \sum_{i \in N} x_i = k,$$

again contradicting feasibility.

No blocking coalition can challenge x . Therefore x is stable. ■

Remark 3 *The key insight underlying this characterization is that stability under single-peaked preferences imposes a highly specific structure on feasible allocations:*

1. *When the total demand of a blocking coalition exceeds the available resources, stability requires that the coalition receives the entire set of goods, leaving nothing to agents outside the coalition, while ensuring that no member receives more than their peak. This implies that there cannot exist two distinct and non trivial (different from the grand coalition) blocking coalitions satisfying this property.*
2. *Conversely, when the total demand of a coalition does not exceed the available resources, stability requires that each member of the coalition receive exactly their peak allocation.*

A consequence of Theorem 2 is stated in the following corollary.

Corollary 2 *Let $(N, \mathcal{B}, R, k)$ be an allocation problem with single-peaked preferences and let $(p_1, \dots, p_n)$ be the collection of all the peaks. If there exists two coalitions $C_1, C_2 \in \mathcal{B}$ with $C_1 \cap C_2 = \emptyset$ such that $\sum_{i \in C_1} p_i > k$ and $\sum_{i \in C_2} p_i > k$, then $\mathcal{C}(N, \mathcal{B}, R, k) = \emptyset$.*

The straightforwardness of Corollary 2 comes from the first condition of Theorem 1 that will give 0 to all of the agents, violating the feasibility of an allocation.

Application: Health-care rationing

A natural application of our framework is the rationing of scarce health-care resources. Public health systems routinely face situations in which the supply of medical resources is limited, while access cannot be allocated solely through market prices. Examples include scarce hospital beds, operating-room slots, treatment courses, or doses of capacity-constrained treatments. In such settings, a central authority must distribute a fixed number k of indivisible and homogeneous units among competing hospitals or regions. Let x_i denote the number of treatment slots allocated to region i , with

$$\sum_{i \in N} x_i = k, \quad x_i \in \{0, \dots, k\}.$$

Each region has preferences over the number of treatment slots it receives, reflecting factors such as the number of patients waiting for treatment, unmet medical needs, capacity constraints, and the opportunity cost of postponing treatment. Let p_i denote the number of slots that region i ideally requires. Under single-peaked preferences, p_i represents the region's effective demand: additional slots improve welfare up to p_i , while allocations beyond this level are less desirable.

The main contribution of our framework is to recognize that, as in many countries, regions may have different institutional power. Health-care rationing is often organized through centralized or multi-level institutions in which different hospitals, regions, professional organizations, or administrative units may possess different capacities to influence the final allocation. Let $\mathcal{B}$ denote the collection of coalitions with sufficient institutional or organizational influence to challenge an allocation. A coalition $C \in \mathcal{B}$ can force the health authority to reconsider an allocation when it can identify a feasible redistribution that makes all its members weakly better off and at least one strictly better off. Thus, an allocation is stable when it is protected against such collectively enforceable challenges. This interpretation shows that health-care rationing is not determined solely by medical need: the distribution of institutional influence also affects which claims can be effectively enforced.

The characterization in Theorem 2 provides a direct interpretation of stability. For any blocking coalition C , two cases arise. If $\sum_{i \in C} p_i > k$, the coalition's aggregate ideal demand exceeds available capacity. Stability then requires that all k units be allocated

within C , with no member receiving more than its peak: $x_j = 0$ for all $j \notin C$, and $x_i \leq p_i$ for all $i \in C$. Conversely, if $\sum_{i \in C} p_i \leq k$, the coalition can be fully satisfied. Stability therefore requires $x_i = p_i$ for all $i \in C$, while the remaining $k - \sum_{i \in C} p_i$ units are allocated outside the coalition.

This result highlights the importance of the influence structure in rationing. If two disjoint influential coalitions C_1 and C_2 both have aggregate demands exceeding capacity,

$$\sum_{i \in C_1} p_i > k, \quad \sum_{i \in C_2} p_i > k, \quad C_1 \cap C_2 = \emptyset,$$

stability would require the entire supply to be allocated to each coalition simultaneously, which is impossible. Hence, no stable allocation exists. The conflict therefore results from the interaction between resource scarcity, heterogeneous demand, and the distribution of influence power.

To illustrate, consider a set of four hospitals $\{1, 2, 3, 4\}$ with ideal demands $(p_1, p_2, p_3, p_4) = (6, 5, 3, 2)$, and suppose hospitals 1 and 2 form the only non trivial influential coalition such that $\mathcal{B} = \{\{1, 2\}, N\}$. If $k = 10$, then $p_1 + p_2 = 11 > 10$. Stability requires all ten slots to be allocated to hospitals 1 and 2, for example $x = (6, 4, 0, 0)$. If capacity increases to $k = 12$, their combined demand can be fully satisfied, so stability requires $x_1 = 6$ and $x_2 = 5$, leaving one slot for another hospital, for example $x = (6, 5, 1, 0)$. Thus, a modest increase in capacity can substantially change the stable allocation.

Finally, suppose hospitals 3 and 4 also form an influential coalition $D = \{3, 4\}$. Since $p_3 + p_4 = 5 \leq 10$, stability with respect to D requires $x_3 = 3$ and $x_4 = 2$, whereas stability with respect to C requires $x_3 = x_4 = 0$. The two requirements are incompatible, so no stable allocation exists. This example shows that scarcity alone does not determine stability; rather, stability depends on how available capacity, ideal demands, and influential coalitions interact.

4.2 Stability with single-dipped preferences

The class of single-dipped preferences was introduced by [Inada \(1964\)](#).³ Roughly speaking, single-dipped preferences arise when each agent has a worst quantity, referred to as their “dip,” and their welfare increases as their assignment moves farther away from this dip. Formally, a preference profile $R = (\succsim_i)_{i \in N}$ is a profile of single-dipped preferences if, for each agent $i \in N$, there exists $d_i \in \{0, \dots, k\}$ such that, for all $q, r \in \{0, \dots, k\}$,

$$r < q \leq d_i \text{ or } d_i \leq q < r \Rightarrow r \succsim_i q.$$

A profile of single-dipped preferences R can then be assimilated to the vector of dips

³Sometimes single-dipped preferences are also referred to as single-caved preferences (e.g., [Diss et al., 2023](#)).

$(d_1, \dots, d_n)$. The following theorem provides a complete characterization of stable allocations under single-dipped preferences.

For any coalition $C \subset N$, let:

- $C_- = \{i \in C : 0 \succ_i k\}$ (agents from C who strictly prefer 0 to k);
- $C_0 = \{i \in C : 0 \sim_i k\}$ (agents from C who are indifferent between 0 and k);
- $C_+ = \{i \in C : k \succ_i 0\}$ (agents from C who strictly prefer k to 0).

Let T_0 be the set of agents $i \in N$ such that $\{C \in \mathcal{B} : i \in C\} = \{N\}$. T_0 stands for the set of agent who do not belong to any non trivial blocking coalition. In other words, T_0 can be viewed as the set of agents who have no influence power.

Lemma 1 *Let $(\mathcal{E}, R) \equiv (N, \mathcal{B}, R, k)$ be an allocation problem where $R = (\succsim_i)_{i \in N}$ is a profile of single-dipped preferences with dips $d_1, \dots, d_n$. If $N_0 \cup N_+ = \emptyset$ (i.e., $N = N_-$), then $\mathcal{C}(\mathcal{E}, R) \neq \emptyset$ if and only if $T_0 \neq \emptyset$.*

Proof. $\Rightarrow$) We proceed by contradiction. Assume that $T_0 = \emptyset$ and there exists a stable allocation x . Since $T_0 = \emptyset$, for every $i \in N$ there exists at least one blocking coalition $C_i \in \mathcal{B}$ containing i such that $C_i \neq N$. As $\sum_{i \in N} x_i = k$, there exists at least one agent i_0 with $x_{i_0} > 0$. Let C_{i_0} be a blocking coalition containing i_0 with $C_{i_0} \neq N$. Then $N \setminus C_{i_0} \neq \emptyset$. Choose $j_0 \notin C_{i_0}$ and define the allocation y by $y_{j_0} = k$ and $y_j = 0$ for all $j \neq j_0$. Since $N = N_-$, every agent strictly prefers 0 to k , so all agents in C_{i_0} are better off within y ; i.e., $y_i \succsim_i x_i$ for all $i \in C_{i_0}$. In particular, $y_{i_0} \succ_{i_0} x_{i_0}$. Since C_{i_0} is a blocking coalition, y challenges x via C_{i_0} , contradicting the stability of x . Hence $\mathcal{C}(\mathcal{E}, R) = \emptyset$.

$\Leftarrow$) Assume that T_0 is not empty. Let $i_0 \in T_0$ and consider the allocation x such that $x_{i_0} = k$ and $x_i = 0$ for all $i \neq i_0$. This allocation is stable because it cannot be challenged by N (the unique blocking coalition containing i_0). ■

Lemma 1 exposes a harsh reality of collective stability. Consider a situation where all agents view the good to be allocated as *undesirable* (for example, nuclear waste, pollution quotas, or troops to be sent to a conflict zone). In this case, every agent prefers to receive 0 rather than k (i.e., $N = N_-$). In such a context, Lemma 1 shows that a stable allocation can exist only if the set T_0 is non-empty. Recall that T_0 is precisely the set of powerless agents. The undesirable good must then be assigned either to a powerful agent outside T_0 or, depending on the configuration, to an agent in T_0 , while all other agents receive 0. Thus, collective stability may require either that a powerful agent bears the entire burden or that a powerless agent is forced to do so. In the latter case, the powerless agent is effectively *sacrificed*.

The following theorem provides a complete characterization of stable allocations for single-dipped preferences.

Theorem 3 Let $(N, \mathcal{B}, R, k)$ be an allocation problem where all agents have single-dipped preferences with the dips $d_1, \dots, d_n$, and let x be an allocation. Then x is stable if and only if the following conditions hold:

(i) If $N_+ \neq \emptyset$, then:

- For every $i \in N_- \cup N_0$, $x_i = 0$.
- For every $j \in N_+$, either $x_j = 0$ or $x_j \succ_j 0$.
- For every blocking coalition C with $C_+ \neq \emptyset$, $\sum_{j \in C_+} x_j = k$.

(ii) If $N_+ = \emptyset$ and $N_0 \neq \emptyset$, then there exists a unique agent $i_0 \in N_0$ such that $x_{i_0} = k$ and $x_i = 0$ for all $i \neq i_0$.

(iii) If $N_+ \cup N_0 = \emptyset$ (i.e., $N = N_-$), then $T_0 \neq \emptyset$, and:

- For every $i \notin T_0$, $x_i = 0$.
- For every $i \in T_0$, either $x_i = k$ or $x_i \succ_i k$.

Proof. We prove both directions.

Part 1 ($\Rightarrow$): Stability implies conditions (i), (ii), (iii). Assume that x is stable.

Case (i): $N_+ \neq \emptyset$.

Step 1: We have $x_i = 0$ for all $i \in N_- \cup N_0$. Indeed, assume that there exists $i_0 \in N_- \cup N_0$ with $x_{i_0} > 0$. Since $i_0 \in N_- \cup N_0$, we have $0 \succ_{i_0} t$ for any t . Take any $j \in N_+$. Consider two subcases:

- If $x_i \leq d_i$ for all $i \in N_+$, we define the allocation y by $y_j = k$ and $y_i = 0$ for all $i \neq j$.
- If there exists $j \in N_+$ with $x_j > d_j$, we define the allocation y as follows: set $y_i = 0$ for all $i \in N_- \cup N_0 \cup \{i \in N_+ : x_i \leq d_i\}$, choose such a j , and set $y_j = x_j + \sum_{i \in N_- \cup N_0} x_i + \sum_{i \in N_+ : x_i \leq d_i} x_i$, with $y_i = x_i$ for all other agents.

In both cases, $\sum_{i \in N} y_i = k$ and $y_i \succ_i x_i$ for all $i \in N$, with a strict preference for j . Therefore, y challenges x via N , contradicting stability.

Step 2: For all $j \in N_+$, either $x_j = 0$ or $x_j \succ_j 0$.

We show first that there exists $j_0 \in N_+$ such that $x_{j_0} > d_{j_0}$.

If all $i \in N_+$ have $x_i \leq d_i$ (and so $0 \succ_i x_i$), select $i_0 \in N_+$, set $y_{i_0} = k$ and $y_i = 0$ for $i \neq i_0$. This is an allocation challenging x allocation via N , contradiction. Hence there exists $j_0 \in N_+$ such that $x_{j_0} > d_{j_0}$.

Now assume by contradiction that there is some $j_1 \in N_+$ with $x_{j_1} \neq 0$ and $0 \succ_{j_1} x_{j_1}$; this implies that $0 < x_{j_1} < k$. Define the allocation y by $y_{j_1} = 0$, $y_{j_0} = x_{j_0} + x_{j_1}$, and $y_i = x_i$ for all $i \notin \{j_0, j_1\}$. The allocation y challenges x via N , which is a contradiction.

Step 3: Influence power constraint. Assume there exists a blocking coalition C with $C_+ \neq \emptyset$ such that $\sum_{j \in C_+} x_j < k$. Choose $j_0 \in C_+$ and define y by $y_{j_0} = x_{j_0} + (k - \sum_{j \in C_+} x_j)$, $y_i = 0$ for $i \notin C$, and $y_j = x_j$ for $j \in C_+ \setminus \{j_0\}$. By Step 1, all $j \in C \setminus C_+$ have $x_j = 0$. Then $y \succsim_j x_j$ for all $i \in C$ with $y_{j_0} \succ_{j_0} x_{j_0}$, which means that y challenges x via C , which contradicts the stability hypothesis.

Case (ii): $N_+ = \emptyset$ and $N_0 \neq \emptyset$.

Let $T = \{i \in N : x_i > 0\}$. Since $\sum_{i \in N} x_i = k$ it is clear that $T \neq \emptyset$. Choose $j \in N_0$.

Suppose $|T| \geq 2$. Then there exists $i_1 \in T$ with $0 < x_{i_1} < k$, so i_1 strictly prefers 0 to x_{i_1} . Define y by $y_j = k$ and $y_i = 0$ for $i \neq j$. Then we have $y_i \succsim_i x_i$ for all $i \in N$, with $y_{i_1} \succ_{i_1} x_{i_1}$, which means that y challenges x via N and a contradiction holds. Hence, $|T| = 1$ and let us set $T = \{i\}$. It comes that $x_i = k$.

If $i \in N_-$, then $0 \succ_i k$ and the same allocation y will challenge x . Thus $i \in N_0$. We conclude that there exists a unique $i_0 \in N_0$ with $x_{i_0} = k$ and $x_i = 0$ for $i \neq i_0$.

Case (iii): $N_+ \cup N_0 = \emptyset$ (i.e.; $N = N_-$).

For any $i \in N$, we have $0 \succ_i k$, so the most preferred outcome is 0.

Step 1: From Lemma 1, it follows that $T_0 \neq \emptyset$. Otherwise, there would exist no stable allocation.

Step 2: Show that for all $i \notin T_0$, $x_i = 0$. Suppose, by contradiction, that there exists $i \in N \setminus T_0$ with $x_i > 0$. Since $T_0 \neq \emptyset$, pick $j \in T_0$. By definition of T_0 , there exists a blocking coalition $C_i \in \mathcal{B}$ containing i such that $C_i \neq N$ (because $i \notin T_0$). Consider the allocation y defined by $y_j = k$ and $y_\ell = 0$ for all $\ell \neq j$. Since $N = N_-$, every agent in C_i strictly prefers 0 to any positive amount. In particular, i receives $y_i = 0$, and since $x_i > 0$, we have $0 \succ_i x_i$. Thus, $y_l \succsim_l x_l$ for all $l \in C_i$ with $y_i \succ_i x_i$. This means that y challenges x via C_i , contradicting stability. Hence $x_i = 0$ for all $i \notin T_0$.

Step 3: Show that for all $i \in T_0$, we have either $x_i = k$ or $x_i \succ_i k$.

Assume by contradiction, that there exists $i_0 \in T_0$ such that $k \succ_{i_0} x_{i_0}$ and $x_{i_0} \neq k$ (so $x_{i_0} < k$). Since $\sum_{i \in N} x_i = k$, this implies that, there exists $i_1 \in N$ such that $0 < x_{i_1} < k$. Since $i_0 \in T_0$, the only blocking coalition containing i_0 is N . Consider the allocation y defined by $y_{i_0} = k$ and $y_\ell = 0$ for all $\ell \neq i_0$. We have $y_i \succsim_i x_i$ for all $i \in N$ with $y_{i_1} \succ_{i_1} x_{i_1}$, which means that y challenges x via N , contradicting stability. Hence, we deduce that for all $i \in T_0$, either $x_i = k$ or $x_i \succ_i k$.

Part 2 ($\Leftarrow$): Conditions (i), (ii), and (iii) imply stability. Assume that the conditions hold.

Case (i): $N_+ \neq \emptyset$.

By condition (i), $x_i = 0$ for all $i \in N_- \cup N_0$; $x_j = 0$ or $x_j \succ_j 0$ for all $j \in N_+$, and for every blocking C with $C_+ \neq \emptyset$, $\sum_{j \in C_+} x_j = k$. Taking $C = N$, we get $\sum_{j \in N_+} x_j = k$, so all goods are with N_+ .

Take any $C \in \mathcal{B}$. If $C_+ = \emptyset$, then $C \subseteq N_- \cup N_0$, and all agents in C already receive

their optimal assignment 0, Thus no challenge may be done via C .

If $C_+ \neq \emptyset$, then $\sum_{j \in C_+} x_j = k$. Since $\sum_{j \in N_+} x_j = k$ and $C_+ \subseteq N_+$, we have $x_j = 0$ for all $j \in N_+ \setminus C_+$. For any challenging allocation y , agents in C must increase their assignment, requiring extra goods, which is impossible as C already contains all agents with positive allocations. So no allocation may challenge x . Therefore x is stable.

Case (ii): $N_+ = \emptyset$ and $N_0 \neq \emptyset$.

Condition (ii) gives a unique $i_0 \in N_0$ with $x_{i_0} = k$, others 0. For any $C \in \mathcal{B}$:

- If $i_0 \notin C$, all agents in C have 0 which is their optimal assignment;
- If $i_0 \in C$, any other allocation would be harmful either for i_0 or for some other agent in C . So no allocation can challenge x via C .

Thus x is stable.

Case (iii): $N_+ \cup N_0 = \emptyset$ (i.e.; $N = N_-$).

By condition (iii), $T_0 \neq \emptyset$ and $x_i = 0$ for $i \notin T_0$. By the same condition (iii), for all $i \in T_0$, either $x_i \succ_i k$ or $x_i = k$.

Pick any $C \in \mathcal{B}$. If $C \neq N$, then for any $i \in T_0$, $i \notin C$, so $C \subseteq N \setminus T_0$ and all the agents in C receive $x_i = 0$, which is their optimal assignment. Hence no allocation may challenge x via C .

If $C = N$, consider the two cases:

- If there exists $i_0 \in T_0$ with $x_{i_0} = k$ (0 to others), then all agents except i_0 have 0, optimal. Any challenging y would need to improve i_0 's welfare by decreasing x_{i_0} , which implies that some other agent must receive and their welfare will decrease. Therefore, no allocation may challenge x via N .
- If for every $j \in T_0$, $x_j \succ_j k$, then all agents in T_0 are on or left of their dip and it is not possible to increase the welfare of some without decreasing that of another one. Therefore, no allocation may challenge x via N .

Hence, we conclude that no allocation may challenge x via any blocking coalition and then x is stable. ■

Theorem 3 reveals that stability under single-dipped preferences is fundamentally driven by a logic of resource concentration. In every case, the stable allocation is highly unequal: the entire resource is either shared exclusively among agents who favor the good (those in N_+), allocated to a single indifferent agent (in N_0), or imposed on a powerless agent (in T_0). By contrast, agents who dislike the good and possess sufficient institutional power to block the allocation systematically receive zero. Stability therefore does not arise from a balanced accommodation of competing interests, but from the concentration of the burden on agents who lack the institutional capacity to resist it, while those endowed with

blocking power are effectively protected from undesirable outcomes. In this sense, our result exposes a fundamental tension between collective stability and distributive fairness: the very institutional mechanisms that prevent destabilizing deviations may generate allocations in which the least powerful agents bear a disproportionate share of the collective burden. Stability, rather than mitigating inequalities in the distribution of the resource or its associated burden, can thus reinforce them. This finding highlights an important challenge for the design of collective decision-making institutions: ensuring resistance to blocking coalitions may come at the expense of protecting the most vulnerable agents.

Corollary 3 *Let $(N, \mathcal{B}, R, k)$ be an allocation problem with single-dipped preferences such that $N_+ \neq \emptyset$ and let x be an allocation. If x is stable, then $\{i \in N : x_i > 0\} \subset \bigcap_{C \in \mathcal{B}: C_+ \neq \emptyset} C_+$.*

Corollary 3 follows directly from the first item of Theorem 3 and provides a necessary condition for stability. Specifically, the set of agents receiving a positive amount of the good must be contained in the “core” of essential claimants, that is, among agents whose support is indispensable for sustaining the allocation against blocking. When this core is empty, no stable allocation exists. Thus, the existence of a stable agreement requires more than the presence of agents who benefit from the good: at least some of these agents must possess sufficient institutional importance to sustain a positive allocation. This result helps explain why stable agreements may be difficult to achieve in collective decision-making environments where many agents have a positive claim on a resource, but none is indispensable to the formation of a blocking coalition. In such settings, the absence of a structurally essential claimant prevents any positive allocation from being protected against deviations, thereby ruling out stability altogether.

Application: Allocation of nuclear-waste management burdens

A natural application of our framework is the allocation of the burden associated with nuclear-waste management across regions. The long-term management of radioactive waste requires authorities to determine where waste should be stored or disposed of. Since hosting such facilities generates environmental, economic, and political costs, regions may strongly oppose assuming this responsibility.

Let x_i denote the number of standardized units of nuclear waste assigned to region i , with

$$\sum_{i \in N} x_i = k, \quad x_i \in \{0, \dots, k\}.$$

Regions may have single-dipped preferences over the amount of waste they are required to manage. The dip d_i represents the quantity that is considered most undesirable, with welfare increasing as the allocation moves away from this quantity. In the most natu-

ral case, nuclear waste is viewed as an undesirable burden, so that every region prefers receiving no waste to receiving the entire quantity, i.e., $N = N^-$.

As in other public-policy settings, regions may differ in their institutional influence. Some may have greater political representation, bargaining capacity, or legal and administrative resources to oppose the location of nuclear-waste facilities. Let $\mathcal{B}$ denote the set of blocking coalitions with sufficient influence to challenge an allocation. Stability therefore depends not only on regional preferences but also on the ability of coalitions to oppose an unfavorable distribution.

Theorem 3 provides a particularly striking conclusion in this setting. When $N = N^-$, a stable allocation exists only if $T_0 \neq \emptyset$, where T_0 is the set of regions that do not belong to any non-trivial blocking coalition. Moreover, every region outside T_0 must receive zero waste, while each region in T_0 must receive either the entire quantity k or an amount it strictly prefers to k . Hence, when all regions regard nuclear waste as undesirable, stability may require concentrating the entire burden on a region with limited institutional influence. If no such region exists, no stable allocation can be achieved.

To illustrate, consider four regions $\{1, 2, 3, 4\}$ and $k = 10$ units of nuclear waste. Suppose that all regions strictly prefer receiving zero units to receiving ten units. Assume that the influence structure is given by $\mathcal{B} = \{\{1, 2\}, \{2, 3\}, N\}$, such that region 4 does not belong to any non-trivial blocking coalition; that is $4 \in T_0$, whereas the other regions possess sufficient institutional influence to participate in such coalitions. The allocation $x = (0, 0, 0, 10)$ assigns the entire waste-management burden to region 4 and zero to the other regions. According to Theorem 3, such an allocation can be stable: the influential regions are protected by their ability to block, while the region without blocking power bears the burden. By contrast, if $T_0 = \emptyset$, Lemma 1 implies that no stable allocation exists.

This example highlights a fundamental tension between stability and distributive fairness. Although every region would prefer to avoid nuclear waste, institutional influence determines which regions can protect themselves from the burden. Stability may therefore be achieved through the concentration of an undesirable burden on the least influential region, showing that collective stability can reinforce existing inequalities in institutional power.

5 Conclusion

This paper studies the allocation of a fixed number of indivisible and homogeneous goods among agents whose institutional influence may be asymmetric. We introduce a notion of stability based on blocking coalitions, which captures the possibility that sufficiently influential groups may challenge an allocation they consider unfavorable. This concept generalizes Pareto optimality and allows us to account for an important feature of many

economic and political environments: agents do not necessarily have the same ability to influence collective decisions.

We first investigate the existence of stable allocations and identify conditions on the influence structure that guarantee stability. We then characterize stable allocations when preferences are single-peaked and single-dipped. Under single-peaked preferences, stability imposes a highly structured allocation pattern determined by the agents' ideal quantities. Under single-dipped preferences, the results reveal a fundamentally different mechanism: stability is typically associated with the concentration of the good, or of an undesirable burden, among a limited number of agents. In particular, when all agents regard the good as undesirable, stability may require assigning the entire burden to an agent with no effective blocking power.

The analysis also suggests several directions for future research. An important extension would be to study richer preference domains and alternative notions of influence, as well as to investigate mechanisms that combine stability with explicit equity or fairness requirements. The compatibility between stability and strategyproofness also raises an interesting question for future research.

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